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MULTIUSER OPTIMAL TRANSMIT BEAMFORMING: PERFORMANCE STUDIES, ANTENNAS SELECTION, A GENETIC ALGORITHM APPROACH

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MULTIUSER OPTIMAL TRANSMIT BEAMFORMING: PERFORMANCE STUDIES, ANTENNAS SELECTION, A GENETIC ALGORITHM APPROACH

présenté par : HOQUE Faika

en vue de l'obtention du diplôme de : Maîtrise ès sciences appliquées

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DEDICATION

To my parents and sisters.....

For their endless love, support and encouragement

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One of the joys of completion is to look over the journey past and remember all the people who have helped and supported me along this long but fulfilling road. The present work would not have been possible without the valuable help of my academic advisors.

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RÉSUMÉ

La formation de faisceaux est une technique très prometteuse utilisant un grand nombre d'antennes pour transmettre un signal vers un ou plusieurs utilisateurs. L'objectif est d'augmenter la puissance du signal chez l'utilisateur souhaité et de réduire la puissance d'interférence chez les utilisateurs non visés. Étant donné que la transmission de la formation de faisceaux augmente la puissance dans une direction spécifique, cela permet à un accès multiple par division spatiale de servir plusieurs utilisateurs simultanément. Cependant, le problème est de garder un équilibre entre maximiser la puissance du signal et minimiser la puissance d'interférence dans les systèmes multi-utilisateurs. Cette thèse décrit une structure simple qui fournit une base théorique pour un système de formation de faisceau optimal. Dans cette thèse, nous étudions les propriétés des systèmes linéaires et optimaux dans différents scénarios, tels que les rapports des signaux faibles et élevés au bruit, des nombres multiple d'antennes, le canal à évanouissement de Rayleigh et les retards multiples. Nous analysons les scénarios lorsque la formation de faisceaux linéaires fonctionnent comme une formation de faisceau optimale. Ensuite, nous proposons une méthode simple pour sélectionner le nombre minimum d'antennes suffisantes pour satisfaire aux exigences de qualité de service des utilisateurs. Lorsque le nombre d’antennes à la station de base est très grand, il ne sera peut-être pas nécessaire d’utiliser toutes les antennes pour desservir seulement quelques utilisateurs. Cette situation incite à choisir un nombre d’antennes limité. Cependant, le nombre choisi peut ne pas suffire à satisfaire les exigences de qualité de service des utilisateurs en raison de fortes interférences, de conditions de canal et du nombre d'utilisateurs. Pour résoudre ce problème NP-difficile, il faut faire une recherche exhaustive ou une recherche heuristique des méthodes itératives avec un coût de complexité informatique acceptable. Ainsi, nous proposons un cadre simple pour sélectionner un ensemble d'antennes suffisantes pour satisfaire les besoins de l'utilisateur. Enfin, nous proposons un algorithme génétique pour une formation de faisceaux optimale avec une complexité d'implémentation faible. Considérant l'algorithme de réduction de branche comme une référence, nous comparons la performance de l'algorithme proposé dans différents scénarios.

ABSTRACT

Transmit beamforming is a very promising technique to transmit the signal from a large array of antennas to one or multiple users. The goal is to increase the signal power at the desired user and reduce the interference power at the non-intended users. Since transmit beamforming increases the power to a specific direction, it allows for space division multiple access to serve multiple users simultaneously. However, the problem is to keep the balance between maximizing the signal power and minimizing the interference power in multi-user systems. This thesis describes a simple structure that provides a theoretical foundation for optimal beamforming scheme. In this thesis, we study the properties of linear and optimal beamforming schemes in different scenarios such as low to high signal to noise ratio ranges, multiple number of antennas, simple Rayleigh fading channel, Rayleigh fading channel with Doppler effects. We analyze the scenarios when linear beamforming performs as an optimal beamforming. Next, we propose a simple method to select the minimum number of antennas that is enough to satisfy the quality of service requirements of the users. In case of massive number of antennas at base station, it may not be necessary to use all antennas to serve only few users. That situation motivates the selection of a set of limited number of antennas. However, the number of chosen antennas may not be enough to satisfy the quality of service requirements of the users due to strong interference, channel conditions and number of users. To solve this NP-hard problem, it requires an exhaustive search or heuristic search, iterative methods with a cost of computational complexity. Thus, we propose a simple framework to select a set of antennas that is enough to satisfy the user's requirements. Finally, we propose a genetic algorithm for optimal beamforming with low implementation complexity. Considering the branch reduce and bound algorithm as a benchmark, we compare the performance of the proposed algorithm in different scenarios.

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LIST OF ACRONYMS AND ABBREVIATIONS

SDMA	Space Division Multiple Access
NP	Non-Deterministic Polynomial
MRT	Maximum Ration Transmission
ZFBF	Zero Force Beamforming
MMSE	Minimum Mean Square Error
MIMO	Multiple Input Multiple Output
QoS	Quality of Service
SNR	Signal to Noise Ratio
SINR	Signal to Interference Noise Ratio
GA	Genetic Algorithm
BRB	Branch Reduce and Bound
SOCP	Second Order Cone Programming
MU-MIMO	Multi-User Multiple Input Multiple Output
SIMO	Single Input Multiple Output
MISO	Multiple Input Single Output
NBS	Norm Based Selection

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CHAPTER 1 INTRODUCTION

1.1 Background and Objective

In wireless communications, the data is sent as electromagnetic waves through the environment (air, buildings, trees etc.) between the devices. The wireless channel distorts the signal, adds interference from other radio signals produced in the same frequency band and adds thermal background noise. As the radio frequency is the global resource for long range applications, wireless communication system should be designed to use the frequency resources as efficiently as possible. The overall efficiency and user satisfaction can be improved by dynamic allocation and management of the available resources. The spectral efficiency can be improved by allowing many devices to communicate in parallel and thus contribute to the total spectral efficiency. Modern multi antenna techniques enable resource allocation with precise spatial separation of users. It is possible to increase the received signal power to the intended user and at same time omit the interference to the other non-intended users by steering the power to a particular direction. The concept of steering the power to a particular direction is called beamforming. Transmit the signal from the multiple antennas using different relative amplitudes and phases such that components ad up constructively in desired directions and destructively in undesired directions. The beamforming resolution depends on the propagation environment and the number of transmit antennas [1]. If there is line of sight (LoS) between the transmitter and receiver, beamforming can be seen a signal beam toward the receiver as showed in Figure 1.1.

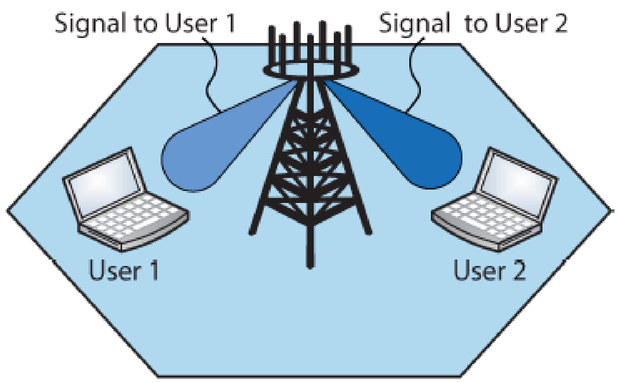


Figure 1.1: Multi-antenna transmission

Beamforming can also be applied in a non-LoS scenario if the multipath channel is known at the transmitter side. Since transmit beamforming focuses the signal energy to a specific place, it allows for multiple users to be served simultaneously. This is called space division multiple access where multiple users are spatially separated. One beamforming vector is assigned to each user and can be matched to its channel. However, the finite number of antennas may not be sufficient for all users which typically lead to leakage of signal power interfering with other users. It is very easy to design a beamforming vector that maximize the signal power to the intended user, but difficult to minimize the interference power. Thus, optimization of multiuser beamforming is a nondeterministic polynomial time (NP) hard problem [2].

In the first section, we study a simple structure of the optimal beamforming [3] with intuitive properties and interpretations. Moreover, we study the properties of linear beamforming schemes known as maximum ratio transmission (MRT), zero force beamforming (ZFBF) and minimum mean square beamforming (MMSE). We study the properties and performance of the transmit beamforming schemes in two types of channel: 1) Rayleigh fading channel assuming the channel is static for many transmitted symbols assuming the users have a fixed location 2) Rayleigh fading channel with real time scenario such as moving users that results in Doppler effects and multipaths delays. In addition, we study the properties of linear and optimal beamforming for two cases: 1) when the number of antennas is equal to the number of users and 2) when the number of antennas is much larger than the number of users.

Next, we study how many antennas we actually need to satisfy the QoS constraints in a massive multiple input multiple output (MIMO) scenario. Multiple antenna wireless communication systems have recently attracted significant attention due to their higher capacity and better tolerance of the fading. Moreover, it allows to reduce the interference power by spatially separating the multiple users. However, increasing the number of transmit antennas enables to improve system performance at the price of higher hardware costs and computational complexity. For a system with large number of antennas arrays, this motivates developing techniques with reduced hardware and computational costs. An efficient approach to achieve this goal is to select the optimal antennas subset. In this thesis, we propose a simple antenna selection method for massive MIMO systems. The method not only selects optimum number of antennas but also guarantee to satisfy the quality of service (QoS) of each user. We perform the proposed method for two types of channel as mentioned earlier and compare the performance for both channel types.

Another important case we study is whether MRT, ZFBF and MMSE are truly optimal. The articles presented in [4], [5] show that MMSE is truly optimal only in special cases. For example, a symmetric scenario where the channels are equally strong and have well separated directivity. In fact, transmit MMSE beamforming performs well and satisfy the optimal beamforming structure in a symmetric scenario. However, ZFBF and MMSE beamforming is not optimal in asymmetric channel conditions. Another case is when users are well separated, and the number of antennas is much larger than the number of users [6]. In general, asymmetric channel conditions and low degree of diversity do not provide enough degree of freedom for MMSE and ZFBF to perform well. In such cases we truly need an optimum beamforming scheme to adjust the user performance. Hence, we propose an efficient algorithm to obtain multiuser optimum beamforming. We propose a genetic algorithm (GA) to find an optimum beamforming. Genetic algorithm is a heuristic search and optimisation technique inspired by natural evolution. We compare the performance of the proposed algorithm with branch reduce and bound (BRB) algorithm considering as the performance benchmark. We also analyze the implementation complexity of the proposed algorithm and show which parameters increase the complexity as well as the performance of the proposed algorithm.

1.2 Chapter Outline

The contributions of this work are folded in three sections:

- Performance studies of the linear beamforming and optimal beamforming in simple fading channels and fading channels with Doppler effects and path delays. Moreover, the effect of very large arrays of antennas on performance metric is reviewed.
- In case of large arrays of antennas, we investigate how the number of unnecessary antennas can be reduced to minimize computational complexity.
- We propose an efficient algorithm to find the optimal beamforming solution with low implementation complexity.

The remainder of this thesis is organized as follows. Chapter two describes the existing works. Chapter three presents the system model, problem formulation and a simple structure of the optimal beamforming. Moreover, the chapter shows the simulation results, our contributions in two different sections and discussions of the results. Chapter four proposes an efficient genetic algorithm for multi user optimum beamforming. Chapter five concludes by summarizing the work done and the main results reported in this thesis.

CHAPTER 2 LITERATURE REVIEW

Multiuser multiple input and multiple output (MIMO) has been extensively studied as one of the key spectral efficiency technologies. Multiple antennas at the base station (BS) enable simultaneous transmissions to multiple users to increase cell capacity. Due to the simplicity of the implementations and near optimal performance, linear beamforming techniques such as maximum ratio transmissions (MRT) [7], [8], zero force beamforming [9], [10], and minimum mean square beamforming (MMSE) [11], [12], are developed for MU-MIMO systems. The following parts explain when and why linear beamforming approaches based on MRT, ZFBF and MMSE are close to optimal and their limitations in different scenarios.

Consider a base station with N antennas communicating with K single antenna user devices. The channel to k^{th} user is represented in the complex baseband by the vector $\boldsymbol{h}_k \in \mathbb{C}^{N\times 1}$. The channel vectors $\{\boldsymbol{h}_k\}_{k=1}^{K}$ is known as the channel state information and is assumed to be perfectly known at the base station. The data signal to user k is denoted $s_k \in \mathbb{C}$ and is normalized to unit power. The K different data signals are separated spatially using the linear beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^{K}$ where $\boldsymbol{w}_k$ is associated with user k. The complex-baseband received signal at k^{th} user is given by the linear input and output model,

$$y_k = \boldsymbol{h}_k^H \left(\sum_{k=1}^{K} \boldsymbol{w}_{\boldsymbol{k}} s_k \right) + n_k \tag{2.1}$$

Where $n_k \in \mathbb{C}$ is the additive receiver noise with zero mean and variance σ^2. The beamforming concept of maximum ratio transmission was introduced in [7] to maximize the signal to noise ratio (SNR) at each user.

$$SNR_k = \frac{p_k}{\sigma_k^2} |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2 \tag{2.2}$$

MRT is the counterpart of maximum ratio combining in receiving process.

$$\boldsymbol{w}_k^{MRT} = \frac{\boldsymbol{h}_k}{\|\boldsymbol{h}_k\|_2} \tag{2.3}$$

MRT can be viewed as a matched filter where the gain of each entry in $\boldsymbol{w}_k^{MRT}$ equals the relative strength of the corresponding channel coefficient in $\boldsymbol{h}_k$ and the phase makes the signal contribution from each channel coefficient add up constructively. The inner product $|\boldsymbol{h}_k^H \boldsymbol{w}_k^{MRT}|$ is therefore maximized which protects the useful signal against channel fading. MRT is the optimal beamforming direction for $K = 1$. However, when there are multiple users, it is not an optimal beamforming because the inter-user interference is uncounted for the MRT beamforming.

Zero forcing refers to signal processing that completely eliminates interference. This can be achieved at the transmitter side by selecting beamforming vectors that are orthogonal to the channels of non-intended users [9]. A theoretical motivation is that zero-forcing simultaneously minimizes the mean square error (MSE) between the received signal and the transmitted symbol. Considering the beamforming matrix $\boldsymbol{W} = [\boldsymbol{w_1}, \dots \dots \boldsymbol{w_K}] \in \mathbb{C}^{\boldsymbol{N \times K}}$ and the channel matrix $\boldsymbol{H} = [\boldsymbol{h_1}, \dots \dots \boldsymbol{h_K}] \in \mathbb{C}^{\boldsymbol{N \times K}}$**,**

$$\boldsymbol{W}^{ZFBF} = \frac{\boldsymbol{H}(\boldsymbol{H}^H \boldsymbol{H})^{-1}}{\|\boldsymbol{H}(\boldsymbol{H}^H \boldsymbol{H})^{-1}\|_2} \tag{2.4}$$

ZFBF is the counterpart of zero-forcing filtering in receive processing. To cancel all inter-user interference, the beamforming directions $\boldsymbol{w}_k^{ZFBF}$ are achieved by projecting the channel vector $\boldsymbol{h}_k$ of the intended user onto the orthogonal complement of the non-intended users. ZFBF provides the optimal beamforming directions at high signal to noise (SNR) regime. Moreover, the loss in signal power due to interference cancelation typically diminishes as the number of transmit antennas increased.

The linear MRT and ZFBF follows from straight extensions of the corresponding criteria for receiver combining such as maximize SNR and minimize the interference power respectively. Wiener filtering balances between signal power maximization and interference power minimization known as minimum mean square error beamforming (MMSE) [10],

$$\boldsymbol{w}_k^{MMSE} = \frac{\left(I_N + \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \boldsymbol{h}_k \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k}{\left\| \left(I_N + \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \boldsymbol{h}_k \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k \right\|_2} \tag{2.5}$$

With the total power P constraint equation (2.5) can be written,

$$\boldsymbol{w}_k^{MMSE} = \frac{\left(I_N + \sum_{k=1}^{K} \frac{P}{K\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k}{\left\| \left(I_N + \sum_{k=1}^{K} \frac{P}{K\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k \right\|_2} \tag{2.6}$$

ZFBF and MMSE beamforming are optimal when the channels are equally strong and the case when $N \gg K$. However, when the channel conditions are varying, and number of antennas are not enough, MMSE and ZFBF can not provide an optimal solution.

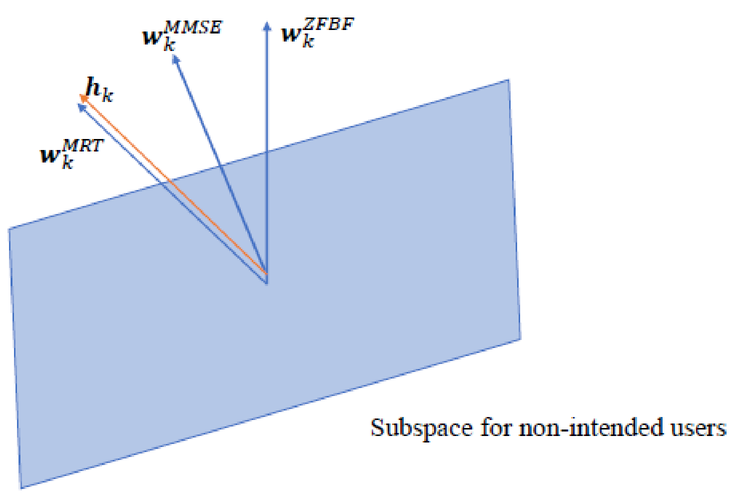


Figure 2.1: Illustration of the beamforming directions with MRT, ZFBF, MMSE beamforming

The assumption of the conventional MMSE beamforming that all users have the same average SNR is clearly invalid in the case of random geometry of the cellular users. A generalized MMSE beamforming which mitigates the practical issues of the conventional MMSE is proposed in [13]. The articles [13] derived the closed form expressions of the generalized MMSE beamforming using convex optimization techniques. The system utilizes the received average SNR that contains the effects of both transmit power and path loss. The problem of joint downlink beamforming in a power-controlled network is proposed in [14], assuming that independent data streams are to be transmitted from a multiantenna base station to several decentralized single-antenna terminals. The total transmit power is limited and channel information (possibly statistical) is available at the transmitter. The design goal is to jointly adjust the beamformers and transmission powers according to individual SINR requirements. In this context, there are two closely related optimization problems. P1: maximize the jointly achievable SINR margin under a total power constraint. P2: minimize the total transmission power while satisfying a set of SINR constraints. In [14], both problems are solved within a unified analytical framework. Problem P1 is solved by minimizing the maximal eigenvalue of an extended crosstalk matrix. The solution provides a necessary and sufficient condition for the feasibility of the SINR requirements. Problem P2 is a variation of problem P1. An iterative strategy is proposed for minimizing the maximal eigenvalue of the extended coupling matrix was derived. The iteration sequence was shown to be monotone and globally convergent.

A general framework for modeling single cell, multi-cell scenarios coordinated beamforming, interference channels, cognitive radio and spectrum sharing is proposed [15]. The performance of multicell systems depends on the resource allocation such as power, frequency and spatial resources are divided among users. The tutorial [15] provides a pragmatic foundation for resource allocation where the system utility metric can be selected to achieve feasibility. Resource allocation is formulated as multi-objective optimization problem and the boundary of the performance region is also represented as efficient solutions. The multi-objective resource allocations problem is solved with poly block outer approximation algorithm. Although the algorithm converges, the worst-case convergence speed is generally exponential in the number of users K. The number of antennas N and power constraints P will however have much smaller impact on the convergence scaling of the PA algorithm, as it approximates the K dimensional performance region. The main computational complexity lies in the bounding procedure which includes a quasi-convex line-search. In practice, it might be necessary to stop the algorithm before it converges. Later, the tutorial also proposed branch reduce and bound algorithm to obtain the optimal solution. This formulation of the algorithm is a slight modification of the algorithm in [16], where the generic BRB algorithm from [17] is adapted for multi-cell resource allocation. Other adaptations are available in [18, 19, 20], where another bounding procedure is used. The system model of [18] is less general than [16], while [19, 20] are limited to single-antenna transmitters but can handle multi-cast transmissions. The convergence of the BRB algorithm to the global optimum was established in [17] and the following theorem originates from [16]. Both algorithms have a worst-case complexity that increases exponentially with the number of users K thus, both algorithms are unsuitable for real-time applications and only practically useful for solving problems with a small number of users.

A more precise and simple structure of optimal beamforming is presented in [3]. The structure provides a theoretical foundation for practical low-complexity beamforming schemes. The lecture shows the properties of linear beamforming schemes such as MRT, ZFBF, transmit MMSE and optimal beamforming based on BRB algorithm. We study the properties of different beamforming schemes based on the beamforming structure presented in [3]. An important observation from that article is when there are many more antennas than users $N \gg K$, it makes the need for optimal beamforming. With a very large arrays where the number of antennas goes high, the linear beamforming schemes that is MMSE and ZFBF schemes performs as same as optimal beamforming in the low to high SNR range.

However, the main limitation of increasing the number of transmit and receive antennas is typically not the number of sensors but the cost of the corresponding RF channels for these antennas and the high complexity required for signal encoding and decoding. This limitation may be more severe when there are some power constraints. A promising way of capturing a large portion of the channel capacity in MIMO systems at reduced hardware costs and computational complexity is to select optimally a small number of “best” antennas from the larger set of antennas available. Antennas selection algorithms [21] [22] [23] are based on channel quality, minimum transmission power consumption, maximum throughput as performance metric has been performed in the previous literatures.

Let consider the number of selected antennas is L. To select the best set of antennas, the channel quality or the performance metrics such as maximum sum throughput, minimum transmission power has to be computed for $\binom{N}{L}$ possible combinations of them. Antenna subset selection has been studied in the literature [24], [30]. Some of these studies focused on system model such as multiple input single output (MISO), single input multiple output (SIMO), multiple input and multiple output (MIMO). In case of SIMO [25] [26], selection has been made based on the signal power considerations for the receive antennas. Norm based selection method has been used MISO environment in [27], [28]. Indeed, the NBS algorithm has a very low complexity of $O(N)$, but it is clarified in [29] that the main drawback of this algorithm is that it may lead to a much lower capacity than that achieved by the optimal selection procedure in scenarios when some rows of the channel matrix are close to be linearly dependent. A promising approach for the fast antenna subset selection was proposed in [29]. This algorithm finds a near-optimal selection of receive antennas based on the capacity maximization. The algorithm begins with the full set of antennas available and then removes one antenna per step. In each step, the antenna with the lowest contribution to the system capacity is removed. The reduction in capacity due to removing of each single antenna is evaluated using a proper updating formula. This process is repeated until the required number of antennas remains. The complexity of this approach is $O(N^2)$. A novel fast near optimal antenna selection algorithm was proposed in [30]. The algorithm starts with empty set of selected antennas and then adds one antenna per step to this set. In each step, the antenna with the highest contribution to the system capacity is adds to the set of selected antennas. Only the receive antennas selection case will be considered until the full set of transmit antennas is used ($L = N$).

The problem of joint multicast beamforming and antenna selection has been addressed in [22]. It is shown that the mixed $l_{1/2}$ norm squared is a prudent group-sparsity inducing convex regularization, in that it naturally yields a suitable semidefinite relaxation to solve the NP hard problem. The paper also indicated that the proposed algorithm significantly reduces the number of antennas required to meet the prescribed quality of service level. The algorithm iteratively run the weighted $l_{1/2}$ norm algorithm to find a solution for the given number of limited antennas L. Then the algorithm solve a semidefinite relaxation programming problem of transmit power minimization problem and use binary search algorithm to find a solution of λ parameter in relaxation problem that gives the required number of antennas L. Then the algorithm use randomization technique to generate candidate sets of beamforming vectors and choose the set that yields a minimum power solution among all candidate sets.

Joint network power minimization and base station selection scheme for cloud RAN is proposed in [31]. The paper proposed a greedy selection algorithm which selects one base station to switch off at each step. To select the base station to be switched off that maximizes the reduction in the network power consumption at each step. However, the greedy selection algorithm induces complexity exponentially with the number of base stations. To further reduce the complexity, the paper proposed three stages group sparse beamforming (GSBF) framework, by adopting the weighted mixed ${}^{l_1}\!/_{l_p}$ norm to induce the group sparsity for the beamformers. By applying the mixed ${}^{l_1}\!/_{l_p}$norm to induce group sparsity, the additional prior information that is transport link power consumption, power amplifier efficiency, and instantaneous effective channel gains to design the weights for different beamformer coefficient groups, resulting in a significant performance gain. Two GSBF algorithms with different complexities are proposed: a) a bi-section GSBF algorithm and b) an iterative GSBF algorithm. Using the weighted mixed l_1/l_2 norm as a replacement for the objective function, the algorithm minimized the weighted mixed l_1/l_2 norm to induce the group sparsity for the beamformer. Then the base stations are switch off under given priorities. The priorities are given with smaller coefficients that is measured by l_∞ norm. Moreover, other system parameters that indicate the priority to be switched off is the channel power gain. Therefore, the channel power gain contributes more to the sum capacity and it provides a higher power gain and should not be encouraged to be switched off. Different from the aggressive strategy in the bi-section GSBF algorithm, which assumes that the RRH should be switched off as many as possible and thus results a minimum transport network power consumption, we adopt a conservative strategy to determine the final active RRH set by realizing that the minimum network power consumption may not be attained when the transport network power consumption is minimized.

The performance of antenna selection-based MIMO networks with large but finite number of antennas and receivers are proposed in [23]. The paper proposed genetic algorithm to select the antennas with sum throughput as objective function and zero forcing precoder at base station. The paper showed that genetic algorithm can be implemented with different objective function and precoding method.

Genetic algorithm has been proposed [38] for MIMO systems to obtain the position and orientation of each MIMO array antenna that maximizes the ergodic capacity for a given propagation scenario. One challenging task in the MIMO system design is to accommodate the multiple antennas in the mobile device without compromising the system capacity, due to spatial and electrical constraints. Based on an interface between the antenna model and the propagation channel model, the ergodic capacity is considered as the objective function of the MIMO array optimization. The goal is to find an optimal or a suboptimal configuration for antenna position and orientation that maximizes the ergodic channel capacity. Assuming an array of dipoles and a channel model that interfaces the propagation environment with the antenna array response pattern, the GA manages to find, for each antenna, the best position and orientation subject to a space constraint. Due to the nature of GAs, the proposed method is very general. It can incorporate different types of antenna models, and it can be also used in different propagation channel models.

In [32], the authors resort to GA-based optimization to find channel parameters such as multipath attenuations and delays. In [33], a GA is used for blind channel estimation. The study reports that the GA method can offer better channel estimation accuracy than traditional methods. A similar approach was also proposed in [34]. Recently, a GA has been used to find good antenna element positions in sparse MIMO radar arrays [35] by minimizing the sidelobes of the radar pattern. Another recent work [36] used a GA to find the optimal distribution of a 3×3 MIMO system for an indoor propagation channel. An interesting aspect of that work is the inclusion of electromagnetic coupling in the model. However, the work does not show either which distributions were found or how the distributions change according to different multipath channel parameters. The work in [37] defends the idea of using nature-inspired methods for MIMO antenna design, but the works mentioned in the proposed work deal with the problem of antenna geometry definition and not antenna array topology for different propagation environments.

Opportunistic beamforming is exploited in independent time-varying channels across multiple users [39]. The diversity benefit is exploited by tracking the channel fluctuations of the users and scheduling transmissions to users when their instantaneous channel quality is near the peak. The diversity gain increases with the dynamic range of the fluctuations and is thus limited in environments with little scattering and/or slow fading. The paper proposed a scheme that induces random fading when the environment has little scattering and/or the fading is slow. Moreover, the article focused on the downlink of a cellular system. The paper used multiple antennas at the base station to transmit the same signal from each antenna modulated by a gain whose phase and magnitude is changing in time in a controlled but pseudorandom fashion. The gains in the different antennas are varied independently. Channel variation is induced through the constructive and destructive addition of signal paths from the multiple transmit antennas to the (single) receive antenna of each user. The overall (time varying) channel signal-to-interference-plus-noise ratio (SINR) is tracked by each user and is fed back to the base station to form a basis for scheduling. The proposed scheme can be viewed as opportunistic beamforming as the transmit powers and phases are randomized and transmission is scheduled to the user which is close to being in the beamforming configuration. The main idea is to amplify the multiuser diversity gain by inducing faster and larger fluctuations.

CHAPTER 3 SYSTEM MODEL AND PROBLEM ANALYSIS

3.1 System Model

Consider a single cell scenario where a base station equipped with N antennas communicates with K user devices, as shown in Figure 3.1. The users are assumed to have single effective antenna. This scenario can be viewed as the superposition of several multiple input single output links (MISO). Thus, it is also known as multi-users MISO communication. The channel to k^{th} user is assumed to be flat fading and represented in the complex baseband by the vector $\boldsymbol{h}_k \in \mathbb{C}^{N\times 1}$. The channel vectors $[\boldsymbol{h}_1, \boldsymbol{h}_2, \dots \dots \boldsymbol{h}_K]$ are non-correlated to each other. The complex valued element $[\boldsymbol{h}_k]_n$ describes the channel from the n^{th} transmit antenna to k^{th} user. Its magnitude represents the gain (or attenuation) of the channel. We assume that the channel vector is quasi-static that is constant for the duration of many transmission symbols. The channel vectors $\{\boldsymbol{h}_k\}_{k=1}^{K}$ is known as the channel state information and is assumed to be perfectly known at the base station. The data signal to user k is denoted $s_k \in \mathbb{C}$ and is normalized to unit power. The K different data signals are separated spatially using the linear beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^{K}$ where $\boldsymbol{w}_k$ is associated with user k. The normalized version $\frac{\boldsymbol{w}_k}{\|\boldsymbol{w}_k\|}$ is called the beamforming direction. The squared norm $\|\boldsymbol{w}_k\|^2$ is the power allocated to the k^{th} user.

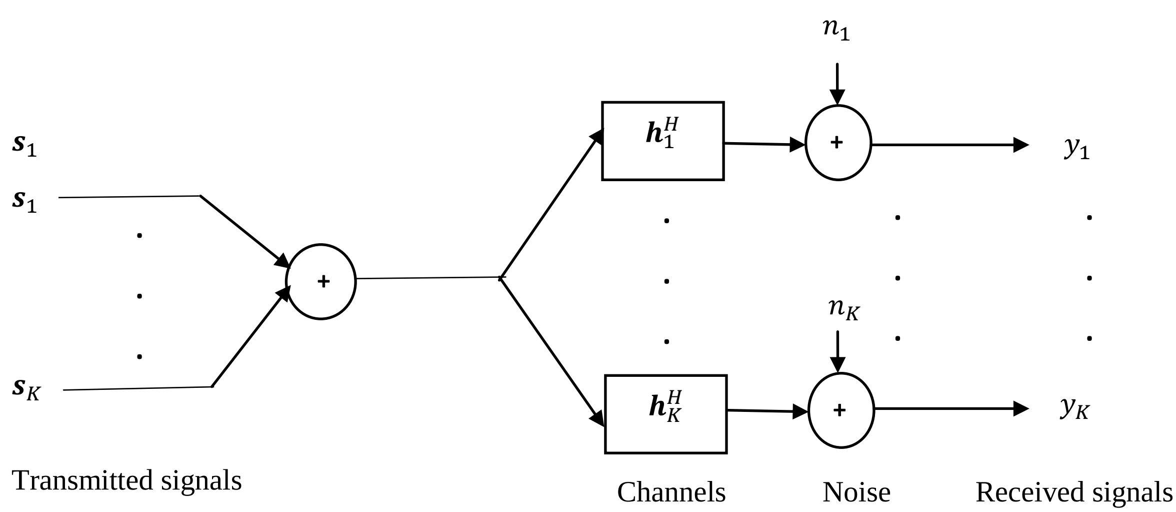


Figure 3.1: Block diagram of the basic system model for a downlink single cell

Under these assumptions, the complex-baseband received signal at k^{th} user is given by the linear input and output model,

$$y_k = \boldsymbol{h}_k^H \left(\sum_{k=1}^{K} \boldsymbol{w_k} s_k \right) + n_k \tag{3.1}$$

Where $n_k \in \mathbb{C}$ is additive receiver noise with zero mean and variance σ^2.

3.2 Resource Allocation Problem

The performance of multi-cell systems depends on the resource allocation such as, how the time, power, frequency, and spatial resources are divided among users. The concept of resource allocation is defined as allocating transmits power among users and spatial directions, while satisfying a set of power constraints that have physical and economic implications. A major complication in resource allocation is the inter-user interference that arises and limits the performance when multiple users are served in parallel. This section formulates the general optimization problem, discusses the solution strategy in later sections, and derives some basic properties of the optimal solution and the performance region.

3.2.1 The Power Constraints

The power resources available for transmission need to be limited somehow to model the inherent restrictions of practical systems. With the total power budget P and the average transmit power $||\boldsymbol{w}_k||^2$ allocated to the k^{th} user, the power constraint can be defined as,

$$\sum_{k=1}^{K} ||\boldsymbol{w}_k||^2 \leq P \tag{3.2}$$

Where P serves as an upper bound on the allowed transmit power in the subspace spanned by $||\boldsymbol{w}_k||^2$. The allocated transmit power $||\boldsymbol{w}_k||^2$ might be the same for all users, but can also be used to define subspaces where the transmit power should be kept below a certain threshold when transmitting to a specific user.

3.2.2 User Performance

To enable low complexity, we assume single user detection that means a user is not attempting to decode and subtract interfering signals while decoding its own signals. This assumption places the responsibility for interference control at the transmitter-side, where the computational resources are available. The corresponding SINR for user k

$$SINR_k = \frac{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{k\neq k'} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + \sigma^2}$$

$$= \frac{\frac{1}{\sigma^2} |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{k\neq k'} \frac{1}{\sigma^2} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + 1} \tag{3.3}$$

The performance is measured by an arbitrary continuous function of $g_k(SINR_k)$ of the SINR. With this definition, it is preferable for k to have a large positive value on $g_k(SINR_k)$ because it corresponds to good performance. Ideally, the function $g_k(.)$ should be selected to quantify the performance quality in a way comprehensible to the user and the system provider. Here, we follow a common example of user performance that is information rate. The achievable information rate is $g_k(SINR_k) = \log_2(1 + SINR_k)$ and describes the number of bits that can be conveyed to user k (per channel use).

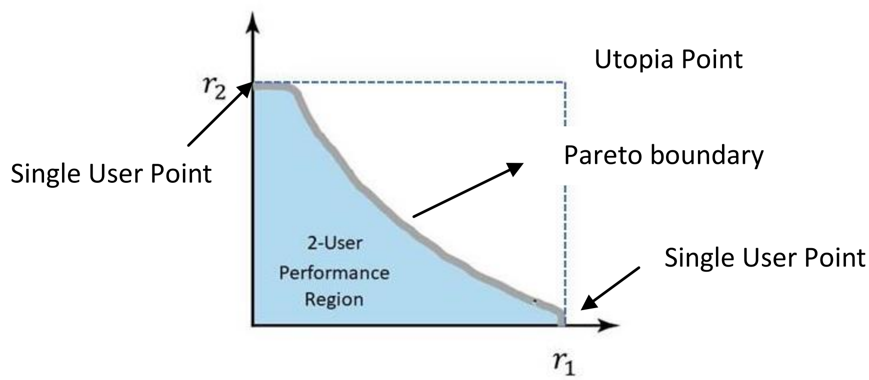
3.2.3 Multi-Objective Resource Allocation

Each user has its own objective $g_k(SINR_k)$ to be optimized, thus there are K different objectives that typically are conflicting. Without loss of generality, our resource allocation problem is formulated as,

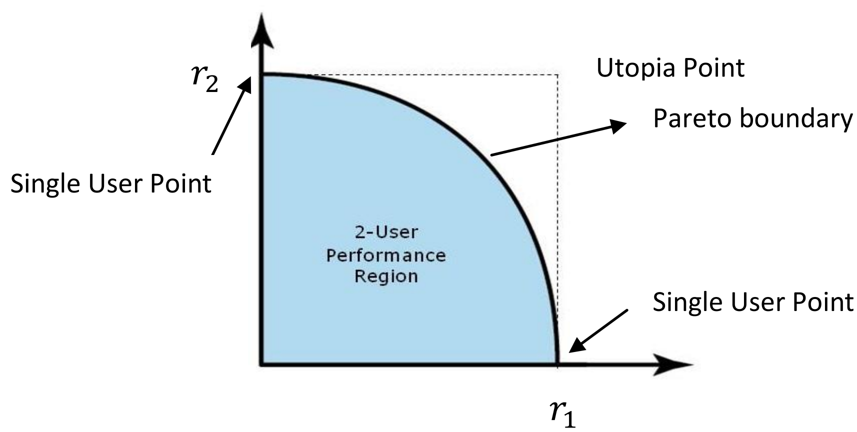
$$\underset{\boldsymbol{w}_1,\ldots\ldots,\boldsymbol{w}_K}{\text{maximize}} \ \{g_1(SINR_1), \ldots\ldots.., g_K(SINR_K)\}$$

$$subject\ to \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \le P \tag{3.4}$$

The optimization problem can be interpreted as searching for a transmit strategy $\boldsymbol{w}_1, \dots\dots, \boldsymbol{w}_K$ that satisfies the power constraints and maximizes the performance $r_1 = g_1(SINR_1)$ of all users. Since the performance of different users is coupled by both power constraints and inter-user interference, there is generally not a single transmit strategy that simultaneously maximizes the performance of all users. For example, $SINR_k$ in (3.3) improves if less interference is caused to k^{th} user but decreasing the interference at k that typically requires decreasing the useful signal power at other users and thereby degrading their SINRs. To study the conflicting objectives of a multi-objective optimization problem, it is instructive to consider the set of all feasible operating points $\boldsymbol{r} = [r_1 \dots \dots . r_K]$ in (3.4) [38], which we call the performance region.



a) Non-Convex and Normal



b) Convex and Normal

Figure 3.2: Examples of compact regions with different shapes

This region describes the performance that can be guaranteed to be simultaneously achievable by the users. The K dimensional performance region is denoted as $\mathcal{R}$ and its shape depends strongly on the channel vectors and power constraints as shown in Figure 3.2

The utopia point **u** is the unique solution to (3.4) in degenerate scenarios when the optimization decouples and all users can achieve maximal performance simultaneously. In general, $\boldsymbol{u} \notin \mathcal{R}$ and represents an unattainable upper bound on performance. There are some tentative solutions in $\mathcal{R}$ that are not dominated by any other feasible point. These points are called Pareto optimal and are such that the performance cannot be improved for any user without decreasing for at least one other user.

We outline the proof from [3] and [15]. For any given $\boldsymbol{r} = (r_1, \dots, r_K) \in \mathcal{R}$, $\boldsymbol{r'} = (r'_1, \dots, r'_K) \in \mathbb{R}_+^K$ with $\boldsymbol{r'} \leq \boldsymbol{r}$ also belongs to $\mathcal{R}$. To this end, let $\boldsymbol{w}_1^*, \dots\dots, \boldsymbol{w}_K^*$ be a feasible transmit strategy that attains $\boldsymbol{r}$ and consider the alternative transmit strategy $p_1\widetilde{\boldsymbol{w}}_1^*, \dots\dots, p_K\widetilde{\boldsymbol{w}}_K^*$, where $p_1, \dots\dots, p_K$ is a set of power allocation coefficients that should belong to

$$\mathcal{A} = \left\{(p_1, \dots\dots, p_K): \sum_{k=1}^{K} p_k \widetilde{\boldsymbol{w}}_k^* \leq P\right\} \tag{3.5}$$

Obviously, the point **r** is achieved by selecting $\boldsymbol{p}_1^*, \dots\dots, \boldsymbol{p}_K^* = (1, \dots\dots, 1)$. To prove that a given $\boldsymbol{r'} \leq \boldsymbol{r}$ is also belongs to $\mathcal{R}$, we need to find $(p_1, \dots\dots, p_K) \in \mathcal{A}$ that gives this point. This corresponds to the conditions $SINR_k = g^{-1}(r'_k) \forall k$, which can be formulated as K linear equations and solved using the approach in [50]. Finally, the existence of $(p_1, \dots\dots, p_K) \in \mathcal{A}$ for any $\boldsymbol{r'} \leq \boldsymbol{r}$ can be proved using interference functions [51].

3.3 Subjective Solutions to Resource Allocation Problem

Recall that the Pareto boundary of the performance region contains all tentative solutions in (3.4), where each representing a certain tradeoff between the users performance. Whenever the utopia points outside of the performance region, there is no objectively optimal resource allocation. To actually compare the merits of different Pareto optimal points, the system designer (or decision maker) needs to bring its own subjective perspective on system utility.

Based on a system utility function, the multi-objective optimization problem in (3.4) can be converted to the following single objective optimization problem (P1),

$$\underset{\boldsymbol{w}_1,\dots\dots,\boldsymbol{w}_K}{\text{maximize}}\, f(g_1(SINR_1), \dots \dots., g_K(SINR_K))$$

$$subject\ to\ \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \ \le P \qquad (3.6)$$

Problem P1 is very hard to solve. The article in [48] proves that equation (3.6) is a NP hard problem for many common utility functions. For example, the sum rate, $f(g_1(SINR_1), \dots \dots., g_K(SINR_K)) = \sum_{k=1}^{K} \log_2(1 + SINR_k)$. This problem has a single (non-unique) solution, because the system utility function resolves the conflicting interests in the multi-object problem. The selection of $f\ (.)$ is therefore very important and should be based on a profound knowledge of performance region $\mathcal{R}$. Moreover, all utility functions are subjective by nature, because each function imposes a certain order of vectors in the performance region. This formulation shows that resource allocation means searching $\boldsymbol{r} = [r_1, \dots \dots, r_K]$ that optimizes system utility.

3.3.1 Convex Optimization for Resource Allocation

In this section we study under which conditions the single objective resource allocation problem in (3.6) is linear, convex. These problems can be solved efficiently using interior point methods. [49], [50]. The problem (3.6) has convex constraints. Therefore, the classification strongly depends on the cost function $f(g_1(SINR_1), \dots \dots \dots, g_K(SINR_K))$ which unfortunately is a complicated function that seems a non-convex. The cost function $f(.)$ depends on the SINRs which are non-convex functions of the beamforming vectors $[\boldsymbol{w}_1, \dots \dots \dots., \boldsymbol{w}_K]$. To pinpoint the main cause of non-convexity, [15] represent the SINRs by auxiliary optimization variables γ_k such that $\gamma_k = SINR_k$, equation (3.6) rewrite as,

$$\underset{\boldsymbol{w}_k\ \gamma_k}{\text{maximize}}\, f(g_1(\gamma_1), \dots \dots \dots, g_K(\gamma_K))$$

$$subject\ to\ |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2 \ge \gamma_k (\textstyle\sum_{k \ne k'} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + \sigma^2)$$

$$\sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \leq P \tag{3.7}$$

The second row of (3.7) represents the auxiliary SINR constraints $\gamma_k \leq SINR_k$ the optimal solution always gives equality in these constraints. The main complication lies in the SINR constraints, because $f(g_1(\gamma_1), \dots \dots, g_K(\gamma_K))$ is a convex function with respect to $(\gamma_1, \dots \dots, \gamma_{k)}$. In other words, it is generally the SINR constraints that prevent (3.7) from being a convex problem. These constraints are non-convex because of the multiplication between γ_k (the SINR value at k^{th} user) and (the inter-user interference caused to k^{th} user). Three approaches to resolve the non-convexity can be envisioned.

(1) Fix the inter-user interference caused to each user

(2) Fix the SINR value at each user

(3) Turn the multiplication into addition by change of variables

None of these approaches can be applied successfully to any resource allocation problem, but they will help identifying special cases when (3.6) has a hidden convex structure and thus can be solved efficiently. The existing work [15], [3] consider the second approach for achieving convex formulations.

3.3.2 Minimize the Transmission Power with Fixed Quality of Service Requirements:

As a preparation toward to solve the (3.6), we first solve the relatively simple power minimization problem as outlined in [3], [15]. Power minimization problem (P2) is formulated as follows,

$$\min_{\boldsymbol{w}_1,\dots,\boldsymbol{w}_K} \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2$$
$$subject\ to\ SINR_k \geq \gamma_k \tag{3.8}$$

Consider the case when the system designer knows exactly which performance each user should be allocated that is the optimal SINR values $SINR_1^*, \dots \dots, SINR_K^*$. Now if we set $\gamma_k = SINR_k^*$, and solve P2 for these particular γ_k parameters, the beamforming vectors that solve P2 will now also solve P1. As described in [3], [15] (3.8) finds beamforming vectors that achieves the SINR values $SINR_1^*, \ldots\ldots, SINR_K^*$. The solution to P1 must satisfy the total power constraint in (3.6), because (3.8) gives the beamforming that achieves the given SINRs using the minimum amount of power. Since the beamforming vectors from P2 are feasible for P1 and achieve the optimal SINR values, they are optimal solution to P1 as well. The difference between the relatively easy P2 and the difficult P1 is that the SINRs are predefined in P2 while we need to find the optimal SINR values along with the beamforming vectors in P1

3.3.3 Solution to Transmission Power Minimization with SINR Constraints

Problem P2 can be reformulated as a convex problem [31]. The cost function $\sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2$ is clearly a convex function of the beamforming vectors. Note the absolute values in SINR constraints in (3.8) make $\boldsymbol{w}_k$ and $e^{j\theta_k}\boldsymbol{w}_k$ completely equivalent for any common phase rotation $\theta_k \in \mathbb{R}$ as in [46]. Without loss of optimality, [ref] exploit this phase ambiguity to rotate the phase such that the inner product $\boldsymbol{h}_k^H \boldsymbol{w}_k$ is real valued and positive. This shows that $\sqrt{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2} = \boldsymbol{h}_k^H \boldsymbol{w}_k \geq 0$. By letting $\Re(.)$ denoting the real part, the constraint $SINR_k \geq \gamma_k$ can be rewritten as

$$\frac{1}{\sigma^2 \gamma_k} |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2 \geq \sum_{k' \neq k} \frac{1}{\sigma^2} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + 1 \leftrightarrow \frac{1}{\sqrt{\sigma^2 \gamma_k}} \Re\left(\boldsymbol{h}_k^H \boldsymbol{w}_k\right) \geq \sqrt{\sum_{k' \neq k} \frac{1}{\sigma^2} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + 1} \tag{3.9}$$

The reformulated SINR constraint in equation (3.9) is a second order cone constraint, which is a convex type of constraint [46] [47].

Optimization theory provides many important properties for the reformulated convex problem. In particularly strong duality and that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for the optimal solution. The strong duality and KKT conditions for P2 play a key role in this solution. To show this, define the Lagrange function of P2 as [46]

$$\mathcal{L}\left(\boldsymbol{w}_1, \ldots\ldots, \boldsymbol{w}_K, \ldots.., \lambda_1, \ldots\ldots \lambda_K\right) = \\ \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 + \sum_{k=1}^{K} \lambda_k \left(\sum_{k \neq k'} \frac{1}{\sigma^2} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + 1 - \frac{1}{\sigma^2 \gamma_k} |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2 \right) \tag{3.10}$$

Where $\lambda_k \geq 0$ is the Lagrange multiplier corresponds to the k^{th} SINR constraint. The dual function is $min_{\boldsymbol{w}_1,\ldots\ldots,\boldsymbol{w}_K}\, \mathcal{L} = \sum_{k=1}^{K} \lambda_k$ and the strong duality implies that it equals the total power $\sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2$ at the optimal solution. KKT conditions which say that $\frac{\partial \mathcal{L}}{\partial \boldsymbol{w}_k} = 0,$ for $k = 1, \ldots, K$ at the optimal solution.

$$\boldsymbol{w}_k + \sum_{k' \neq k} \frac{\lambda_{k'}}{\sigma^2} \boldsymbol{h}_{k'} \boldsymbol{h}_{k'}^H \boldsymbol{w}_k - \frac{\lambda_k}{\gamma_k \sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \boldsymbol{w}_k = 0 \tag{3.11}$$

$$\leftrightarrow \left(\mathbf{I}_N + \sum_{k=1} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \right) \boldsymbol{w}_k = \frac{\lambda_k}{\sigma^2} \left(1 + \frac{1}{\gamma_k}\right) \boldsymbol{h}_k \boldsymbol{h}_k^H \boldsymbol{w}_k \tag{3.12}$$

$$\leftrightarrow \boldsymbol{w}_k = \left(\mathbf{I}_N + \sum_{k=1}^{K} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \right)^{-1} \boldsymbol{h}_k \underbrace{\frac{\lambda_k}{\sigma^2} \left(1 + \frac{1}{\gamma_k}\right) \boldsymbol{h}_k^H \boldsymbol{w}_k}_{\text{Scalar}} \tag{3.13}$$

Where $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. The expression (3.13) is achieved from (3.12) by adding the term $\frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \boldsymbol{w}_k$ to both sides. Since $\frac{\lambda_k}{\sigma^2} \left(1 + \frac{1}{\gamma_k}\right) \boldsymbol{h}_k^H \boldsymbol{w}_k$ is a scalar, equation (3.13) shows that optimal solution $\boldsymbol{w}_k$ must be parallel to $\left(\mathbf{I}_N + \sum_{k=1}^{K} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \right)^{-1} \boldsymbol{h}_k$. In other words, the optimal beamforming vectors $\boldsymbol{w}_1^*, \ldots \ldots, \boldsymbol{w}_K^*$ are

$$\boldsymbol{w}_k^* = \underbrace{\sqrt{p_k}}_{\text{Beamforming power}} \underbrace{\frac{\left(\mathbf{I}_N + \sum_{k=1}^{K} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \right)^{-1} \boldsymbol{h}_k}{\left\| \left(\mathbf{I}_N + \sum_{k=1}^{K} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H \right)^{-1} \boldsymbol{h}_k \right\|}}_{\widetilde{\boldsymbol{w}}_k^* = \text{Beamforming direction}} \quad \text{for } k = 1, \ldots., K \tag{3.14}$$

Where p_k denotes the beamforming power and $\widetilde{\boldsymbol{w}}_k^*$ denotes the unit norm beamforming direction for user k. The K unknown beamforming power are computed by solving the SINR constraints in P2. This implies $\frac{1}{\gamma_k} p_k |\boldsymbol{h}_k^H \widetilde{\boldsymbol{w}}_k^*|^2 - \sum_{k' \neq k} p_{k'} \left|\boldsymbol{h}_k^H \widetilde{\boldsymbol{w}}_{k'}^*\right|^2 = \sigma^2$ for k = 1,….., K. Since the beamforming directions are known, we have K linear equations and obtain the K powers as

$$\begin{bmatrix} p_1 \\ \vdots \\ p_K \end{bmatrix} = X^{-1} \begin{bmatrix} \sigma^2 \\ \vdots \\ \sigma^2 \end{bmatrix} \quad where\ [X]_{ij} = \begin{cases} \frac{1}{\gamma_i} \left| \boldsymbol{h}_i^H \widetilde{\boldsymbol{w}}_i^* \right|^2, & i = j \\ -\left| \boldsymbol{h}_i^H \widetilde{\boldsymbol{w}}_j^* \right|^2, & i \neq j \end{cases} \tag{3.15}$$

Where $[X]_{ij}$ denotes the $(i,j)\ th$ element of the matrix $X \in R^{K \times K}$. From equation (3.5) and (3.10), we obtain the structure of optimal beamforming as a function of Lagrange multipliers $\lambda_1, \dots \dots, \lambda_K$. Lagrange multiplier can be computed by convex optimization [46] or from the fixed point equations $\lambda_k = \frac{\sigma^2}{\left(1+\frac{1}{\gamma_k}\right) \boldsymbol{h}_k^H \left(\mathbf{I}_N + \sum_{k=1}^{K} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H\right)^{-1} \boldsymbol{h}_k}$ for all k [46], [47].

3.4 Properties of Beamforming Structure

For some positive parameters $\lambda_1, \dots \dots, \lambda_K$, the strong duality property of P2 implies $\sum_{k=1}^{K} \lambda_k = P$, since P is the optimal cost function in P2 and k=1$K\lambda k$ is the dual function. Since the matrix inverse in (3.15) is same for all users, the matrix $\boldsymbol{W}^* = [\boldsymbol{w}_1^* \dots \dots \dots \boldsymbol{w}_K^*] \in C^{N \times K}$ with the optimal beamforming vectors can be written in a compact form. To this end, we note that $\sum_{k=1}^{K} \frac{\lambda_k}{\sigma^2} \boldsymbol{h}_k \boldsymbol{h}_k^H = \frac{1}{\sigma^2}\ \mathbf{H\Lambda H}^{\boldsymbol{H}}$ where $\mathbf{H} = [\boldsymbol{h}_1 \dots \dots \dots \boldsymbol{h}_K] \in C^{N \times K}$ contains the channels and $\boldsymbol{\Lambda} = \mathrm{diag}\,(\lambda_1, \dots, \lambda_K)$ is a diagonal matrix with the λ parameters. By gathering the power allocation in matrix $\mathbf{P}$, we obtain
the compact equation

$$\mathbf{W}^* = \left(\mathbf{I}_N + \frac{1}{\sigma^2}\ \mathbf{H\Lambda H}^{\boldsymbol{H}}\right)^{-1} \mathbf{HP}^{\frac{1}{2}} \tag{3.16}$$

Where $\boldsymbol{P} = diag\ \left(p_1 / \left\| \left(\mathbf{I}_N + \frac{1}{\sigma^2}\ \mathbf{H\Lambda H}^{\boldsymbol{H}}\right)^{-1} \boldsymbol{h}_1 \right\|^2, \dots, p_K / \left\| (\mathbf{I}_N + \frac{1}{\sigma^2}\ \mathbf{H\Lambda H}^{\boldsymbol{H}})^{-1} \boldsymbol{h}_K \right\|^2\right)$ and $(.)^{\frac{1}{2}}$ denotes the matrix square root.

In the low signal to noise ratio case, represented by $\sigma^2 \to \infty$, the system is noise-limited and the beamforming matrix in (3.16) converges at

$$\mathbf{W}^*_{\sigma^2 \to \infty} = (\mathbf{I}_{\boldsymbol{N}} + \mathbf{0})^{-1}\ \mathbf{HP}^{\frac{1}{2}}_{\sigma^2 \to \infty} = \mathbf{HP}^{\frac{1}{2}}_{\sigma^2 \to \infty} \tag{3.17}$$

Where the matrix inverse vanishes and $\mathbf{P}_{\sigma^2\to\infty}$ denotes the asymptotic power allocation. This implies $\mathbf{w}_k^*$ is scaled version of channel vector $\boldsymbol{h}_k$, which is similar to MRT.

On the other hand, at high signal to noise ratio case, represented by $\sigma^2 \to 0$, the system is interference limited. To avoid the singularity in the inverse when σ^2 is small, we use the identity $(\mathbf{I}+\mathbf{AB})^{-1}\mathbf{A} = \mathbf{A}(\mathbf{I}+\mathbf{BA})^{-1}$ and rewrite as

$$\begin{aligned}\mathbf{W}^*_{\sigma^2\to 0} &= \mathbf{H}(0\mathbf{I}_{\boldsymbol{N}} + \mathbf{\Lambda}\mathbf{H}^{\boldsymbol{H}}\mathbf{H})^{-1}\,\breve{\mathbf{P}}^{\frac{1}{2}}_{\sigma^2\to 0} \\ &= \mathbf{H}(\mathbf{H}^{\boldsymbol{H}}\mathbf{H})^{-1}\;\mathbf{\Lambda}^{-1}\breve{\mathbf{P}}^{\frac{1}{2}}_{\sigma^2\to 0}\end{aligned} \tag{3.18}$$

where the term $0\mathbf{I}_{\boldsymbol{N}}$ vanishes when $\sigma^2 \to 0$ and $\breve{\mathbf{P}}_{\sigma^2\to 0}$ denotes the asymptotic power allocation and $\breve{\mathbf{P}} = \;diag\;(p_1/\left\|(\boldsymbol{\sigma}^2\mathbf{I}_N \;+\; \mathbf{\Lambda}\mathbf{H}^{\boldsymbol{H}}\mathbf{H})^{-1}\boldsymbol{h}_1\right\|^2, \dots, p_K/\left\|(\boldsymbol{\sigma}^2\mathbf{I}_N \;+\; \mathbf{\Lambda}\mathbf{H}^{\boldsymbol{H}}\mathbf{H})^{-1}\boldsymbol{h}_K\right\|^2)$. The solution in (3.15) known as channel inversion or zero forcing beamforming because it contains the pseudo inverse $\mathbf{H}(\mathbf{H}^{\boldsymbol{H}}\mathbf{H})^{-1}$ of the channel matrix $\mathbf{H}^H$. Hence, $\mathbf{H}^H\mathbf{W}^*_{\sigma^2\to 0} = \mathbf{\Lambda}^{-1}\breve{\mathbf{P}}^{\frac{1}{2}}_{\sigma^2\to 0}$ is a diagonal matrix. Since the off-diagonal elements are of the form $\boldsymbol{h}_{k'}^H\boldsymbol{w}_k^* = 0$ for $k' \neq k$, the beamforming causes zero interference by projecting $\boldsymbol{h}_k$ onto the subspace that is orthogonal to the co-user channels.

3.5 Branch Reduce and Bound Approach to Optimization of Resource Allocation

The resource allocation problems have important property that the optimum lies on the Pareto boundary of $\mathcal{R}$. This property should certainly be utilized when devising a numerical algorithm for solving the problem. The naive approach would be to generate a large set of Pareto optimal points, preferably by some approach that finds Pareto optimal points with polynomial computational complexity. However, there are more intelligent and systematic algorithms than this naive approach. These algorithms concentrate on searching parts of the Pareto boundary that give large values on $f(.)$.

This section describes a general algorithm for solving resource allocation the branch-reduce-and-bound (BRB) algorithm from [15]. Algorithm is designed to iteratively improve a lower bound f_{min} and an upper bound f_{max} on the optimal value of (3.6). Convergence to the global optimum will be guaranteed in the sense that

$$f_{min} - f_{max} < \varepsilon \tag{3.19}$$

is achieved in finitely many iterations, for any accuracy $\varepsilon > 0$. In general, the number of iterations scales exponentially with the number of users K, which is an inescapable consequence of solving a problem that generally is NP hard (3.8).

3.5.1 Lower and Upper Bounds in a Box

An essential step in the BRB algorithms is that of bounding the highest feasible performance in a box $\mathcal{M} = [a, b] \subseteq \mathbb{R}_K^+$. This means finding a lower bound $f_{min,M}$ and a upper bound $f_{max,M}$ on the optimal solution. These bounds represent the performance in the lower and upper corners of the box, but only if the box has a nonempty overlap with the performance region, this is equivalent to $a \in \mathcal{R}$ which is easily checked by solving the feasibility problem (3.8) with **a** as the QoS requirements.

3.5.2 BRB Algorithm

The BRB algorithm maintains a set $\mathcal{N}$ with non overlapping boxes that surely covers the parts of the performance region $\mathcal{R}$ where the optimal solutions lie. Iteratively, the algorithm is split into certain boxes and bounds the performance in these new boxes for the purpose of improving a lower bound f_{min} and an upper bound f_{max} on the optimal value of (3.6). To aid this process, a local feasible point g_M and a local upper bound $\beta(\mathcal{M})$ are stored for each box $\mathcal{M} \in \mathcal{N}$. .

Initially, $\mathcal{N} = \{\mathcal{M}_0\}$ or a box $\mathcal{M}_0 = [0, b_0] \subseteq \mathbb{R}_K^+$, where b_0 could be the utopia point $\boldsymbol{u}$ or some other optimistic point that guarantees $\mathcal{R} \subseteq \mathcal{M}_0$. The initial upper bound is $f_{max} = f(b_0)$, while the lower bound is initialized as $f_{min} = f(\boldsymbol{g}_{feasible})$ for some known feasible point.

Each iteration of the BRB algorithm consists of three steps:

1) Branch : Divide a box $\mathcal{M}_{max} \in \mathcal{N}$ into two new boxes
2) Reduce : Remove parts of these new boxes that cannot contain optimal solutions
3) Bound : Apply the bounding procedure to one of the new boxes to improve local and global solutions.

Branch: First, $\mathcal{M}_{max}$ is divided into two disjoint boxes $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$. $\mathcal{M}_{max}$ is bisected along its longest side which produces,

$$\widetilde{\mathcal{M}}_1 = [a_{max}, b_{max} - d\mathbf{e}_{\text{dim}}]$$
$$\widetilde{\mathcal{M}}_2 = [a_{max} + d\mathbf{e}_{\text{dim}}, b_{max}] \tag{3.20}$$

where $\dim = argmax_k[b_{max} - a_{\max}]_k$, $d = [a_{max} + b_{\max}]_{dim/2}$, and $\boldsymbol{e}_k$ is the kth column of the identify matrix $\boldsymbol{I}_K$. The local feasible points and upper bounds of $\widetilde{\mathcal{M}}_1$, $\widetilde{\mathcal{M}}_2$ can be selected as,

$$\boldsymbol{g}_{\widetilde{\mathcal{M}}_1} = \begin{Bmatrix} \boldsymbol{g\mathcal{M}}_{max} - [\boldsymbol{g\mathcal{M}}_{max} - (b_{max} - d\mathrm{e}_{\text{dim}})], & \boldsymbol{g\mathcal{M}}_{max} \geq a_{max} + d\mathrm{e}_{\text{dim}} \\ \boldsymbol{g\mathcal{M}}_{max}, & otherwise \end{Bmatrix}$$
$$\boldsymbol{g}_{\widetilde{\mathcal{M}}_2} = \boldsymbol{g\mathcal{M}}_{max}, \tag{3.21}$$

and the local upper bounds can be selected as

$$\beta\big(\widetilde{\mathcal{M}}_1\big) = \min(\beta(\boldsymbol{\mathcal{M}}_{max}), f(b_{max} - d\mathrm{e}_{\text{dim}})),$$
$$\beta\big(\widetilde{\mathcal{M}}_2\big) = \beta(\boldsymbol{\mathcal{M}}_{max})$$

Reduce : Next, the new boxes $\widetilde{\mathcal{M}}_l = [\tilde{a}_l, \tilde{b}_l]$ for $(l = 1, 2)$ are reduced by removing parts that cannot contain the optimal solution that is, parts that either give performance below the lower bound f_{min} or above the local upper bound $\beta\big(\widetilde{\mathcal{M}}_l\big)$. If $f_{min} > \beta\big(\widetilde{\mathcal{M}}_l\big)$, then $\widetilde{\mathcal{M}}_l$ will not contain the optimal solution and can be removed. Otherwise, all points $\boldsymbol{g} \in [\tilde{a}_l, \tilde{b}_l]$ satisfying $f_{min} \leq f(\boldsymbol{g}) \leq \beta\big(\widetilde{\mathcal{M}}_l\big)$ are also contained in $\big[\tilde{a}'_l, \tilde{b}'_l\big] \subseteq \big[\tilde{a}_l, \tilde{b}_l\big]$, where

$$\tilde{a}'_l = \tilde{b}_l - \sum_{k=1}^{K} v_{lk}[\tilde{b}_{lk} - \tilde{a}_{lk}]\boldsymbol{e}_k$$
$$\tilde{b}'_l = \tilde{a}'_l + \sum_{k=1}^{K} u_{lk}[\tilde{b}_{lk} - \tilde{a}'_{lk}]\boldsymbol{e}_k$$
$$v_{lk} = \max\{v: 0 \leq v \leq 1, f(\tilde{b}_l - v[\tilde{b}_k - \tilde{a}_k]\boldsymbol{e}_k) \geq f_{min}\}$$
$$u_{lk} = \max\{u: 0 \leq u \leq 1, f(\tilde{a}'_l + u[\tilde{b}_k - \tilde{a}'_{lk}]\boldsymbol{e}_k) \leq \beta\big(\widetilde{\mathcal{M}}_l\big)\}$$

Bound: Each iteration ends by a search for better bounds. First, it is checked if there are any feasible points in$\widetilde{\mathcal{M}}_l = \big[\tilde{a}'_l, \tilde{b}'_l\big]$, or if $\widetilde{\mathcal{M}}_l \cap \mathcal{R} = \emptyset$.

The intersection $\widetilde{\mathcal{M}}_l \cap \mathcal{R} = \emptyset$ if $\boldsymbol{g}_{\widetilde{\mathcal{M}}_l} \geq \tilde{a}'_l$. Otherwise, the existence of feasible points in $\widetilde{\mathcal{M}}_l$ can be checked by solving the feasibility problem (3.8) with $\tilde{a}'_l$ as the QoS requirements. If $\widetilde{\mathcal{M}}_2 \cap \mathcal{R} \neq \emptyset$, the BRB algorithm applies the bounding procedure as following

$$\boldsymbol{r}(\tau) = \tilde{a}'_2 + \tau \frac{(\tilde{b}'_2 - \tilde{a}'_2)}{\left\|\tilde{b}'_2 - \tilde{a}'_2\right\|_{\mathbf{1}}} \qquad \tau \in \left[0, \left\|\tilde{b}'_2 - \tilde{a}'_2\right\|_1\right] \tag{3.22}$$

as the search curve and using line search accuracy δ. The normalization $\left\|\tilde{b}'_2 - \tilde{a}'_2\right\|_1$ ensures that the line search accuracy is a global measure, thus the bounding procedure becomes faster as the boxes get smaller.

Finally, the global lower bound is updated as $f_{min} = max_{\mathcal{M}\in\mathcal{N}} f(\boldsymbol{g}\mathcal{M})$ and the global upper bound is updated as $f_{max} = max_{\mathcal{M}\in\mathcal{N}} \beta(\mathcal{M})$. The stopping criterion $f_{max} - f_{min} < \varepsilon$ is checked at the end of each iteration.

3.6 Heuristic Beamforming

A classic scenario in signal processing is the detection of a scalar data symbol $\boldsymbol{s}_k$ which is observed under channel distortion, additive interference and white noise. If multiple channel observations are available for a certain data symbol, the scenario can be written as,

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{h}_k \boldsymbol{s}_k + \boldsymbol{n} \tag{3.23}$$

where $\boldsymbol{h}_k$ is the channel symbol $\boldsymbol{s}_k$, $\mathbb{E}\{s_k\} = 0$, $\mathbb{E}\{|s_k|^2\} = 1$, $\mathbb{E}\{\boldsymbol{n}\boldsymbol{n}^H\} = \sigma^2\boldsymbol{I}$. The symbol can be estimated from the vector valued observation $\boldsymbol{y}$ as $\hat{\boldsymbol{s}}_k = \overline{\boldsymbol{w}}^H\boldsymbol{y}$ using a linear received combining filter $\overline{\boldsymbol{w}}$.

Three classic receive combining techniques are:

(1) Maximum ratio combining: Weighs and aligns the observations as $\overline{\boldsymbol{w}} = \frac{1}{\|\boldsymbol{h}_k\|_2^2 + \sigma^2} \boldsymbol{h}_k$ to maximize the ratio between received signal power and noise power.

(2) Zero forcing filtering: Removes interference by projecting the observations as $\overline{\boldsymbol{w}} = (\sum_{k=1}^{K} \boldsymbol{h}_k \boldsymbol{h}_k^H)^\dagger \boldsymbol{h}_k$, which is the orthogonal complement of the interfering signals. This maximizes the ratio between received signal power and interference power.

(3) MMSE filtering: The mean square error (MSE) minimizing $\bar{\boldsymbol{w}} = (\sum_{k=1}^{K} \boldsymbol{h}_k \boldsymbol{h}_k^H + \sigma 2\boldsymbol{I}) - 1\boldsymbol{h}k$ that balances between maximizing signal power and suppressing interference.

3.6.1 Power Allocation

The previous subsection defined MRT, ZFBF and MMSE as heuristic ways of selecting the beamforming directions $\{\bar{\boldsymbol{w}}_k\}_{k=1}^{K}$. When these have been selected, the power allocation $\{p_k\}_{k=1}^{K}$ ultimately determine the operating point in the performance region that is achieved by the heuristic transmit strategy. For given $\{\bar{\boldsymbol{w}}_k\}_{k=1}^{K}$, the SINR become

$$SINR_k = \frac{p_k \rho_{kk}}{\sigma_k^2 + \sum_{k' \neq k} p_{k'} \rho_{k'k}} \tag{3.24}$$

with fixed $\rho_{k'k} = |\boldsymbol{h}_k^H \bar{\boldsymbol{w}}_{k'}|^2$ for all k, k'. The power allocations can be formulated by so called waterfilling solutions

$$p_k = \left(\frac{1}{\lambda} - \frac{\sigma_k^2}{\rho_{k'k}}\right)^{+} \tag{3.25}$$

where λ is the Lagrange multiplier.

3.7 Performance Studies in Fading Channel

In this section, we have studied the performance of MRT, ZFBF, MMSE and optimal beamforming based on BRB algorithm in two types channel: 1) Rayleigh fading channel assuming that channel is static as receivers are not moving 2) Rayleigh fading channel with proper effects due to moving receivers.

3.7.1 Simulation Environment

To solve the second order cone program (SOCP), CVX is used. CVX is a MATLAB-based modeling system for convex optimization problem [44].

a) **What is CVX**: CVX is a Matlab-based modeling system for convex optimization. CVX turns Matlab into a modeling language, allowing constraints and objectives to be specified using standard Matlab expression syntax. Structure of convex problem,

$$minimize \quad f_0(x)$$

$$subject\ to \quad f_i(x) \leq 0, \quad i = 1, \ldots p$$

In CVX,

```
cvx_begin
   variables x(n)
   minimize (f0(x))
   subject to
      f(x)<=0
cvx_end
```

where f_0 and f_i must be convex.

Return values: Upon exit, CVX sets the variables

- x – solution variables (s) x^*
- cvx_optval – the optimal value p^*
- cvx_status – solver status (Solved, Unbounded, Infeasible)

CVX uses SeDuMi, a MATLAB implementation of second order interior point methods for the actual computations [45]. The algorithms are tested with two channel types: Rayleigh fading channel, $\boldsymbol{h} \sim CN(0, I_N)$ and a Rayleigh fading channel with Doppler effects. Throughout this section, the noise variance for all users is set to $\sigma^2 = 1$.The results are obtained for 100 different Rayleigh channel realizations, $\boldsymbol{h}_k \sim CN(0, I_N)$ and the SNR is measured as $\frac{P}{\sigma^2}$.

BRB algorithm is built upon solving a series of convex feasibility problems with QoS requirements. To achieve certain accuracy ε = 0.01 on the optimal solution, the average number of such feasibility evaluations is 3000 and the number of iterations is 2000 [15]

3.7.2 Rayleigh Fading Channel: Sum rate performance measurement

a) In case users K=4, N=4:

The properties of MRT, ZFBF, MMSE and optimal beamforming are illustrated by simulation in Figure 3.3 We consider K=4 users for this scenario and evaluate the sum rate as utility function: $f(SINR_1, \dots \dots SINR_K) = \sum_{k=1}^{K} log_2(1 + SINR_k)$. Figure 3.3 shows the simulation results for the case N=4 where MRT performs near-optimal at low SNRs, while ZFBF is optimal at high SNRs. MMSE beamforming combines the respective asymptotic properties of MRT and ZFBF with good performance at entire SNRs range. However, there is still significant gap to the optimal solution which is computed by the branch reduce and bound (BRB) algorithm whose computational complexity grows exponentially with K. The significant performance of optimal beamforming is obtained by adjusting the K=4 parameters $\lambda_1, \dots \dots . \lambda_K$.

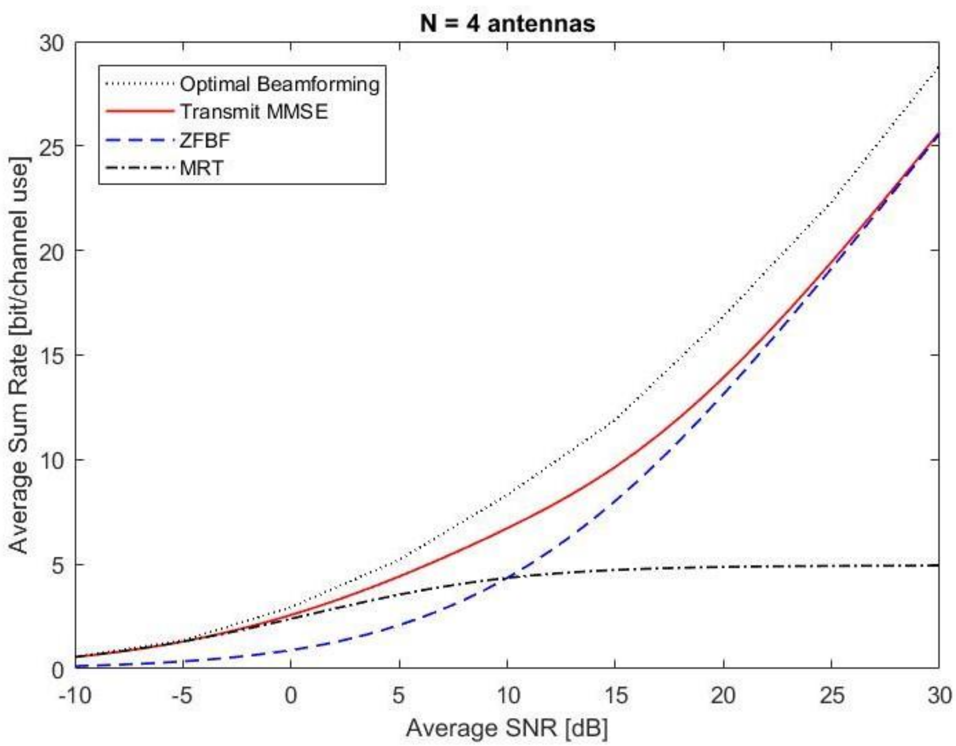


Figure 3.3: Average sum rate vs. SNR for N=4, K=4

b) In case users K=4, N=12:

Massive MIMO has been received much attention to increase the sum throughput. A key motivation is that squared channel norms ($\|\boldsymbol{h}_k\|^2$) are proportional to N, while the cross products $\left(\left|\boldsymbol{h}_{k'}^H \boldsymbol{h}_k\right| \text{ for } k' \neq k\right)$ increase more slowly with N. Hence the user channel becomes orthogonal as $N \rightarrow \infty$, which reduces interference and allows for less transmit power. Note that $(\boldsymbol{\sigma}^2 \mathbf{I}_N + \boldsymbol{\Lambda}\mathbf{H}^H\mathbf{H})^{-1} \approx (\boldsymbol{\Lambda}\mathbf{H}^H\mathbf{H})^{-1}$ for large N, since only the elements $\mathbf{H}^H\mathbf{H}$ grow with N.

Figure 3.4 shows the case when number of antennas are larger than the users, $N \gg K$. Since there are many more antennas than the users, the need for the optimality becomes much smaller.

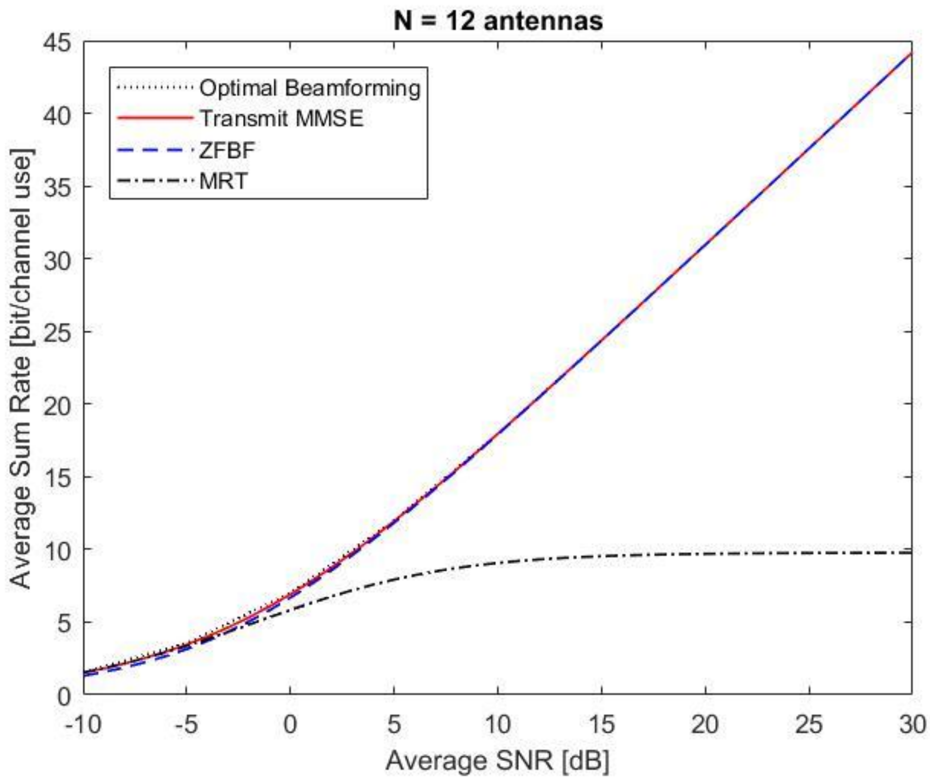


Figure 3.4: Average sum rate vs. SNR for N=12, K=4

An important observation is transmitting MMSE beamforming and ZFBF is almost same as optimal beamforming for a system with very large antennas.

3.7.3. Rayleigh Fading Channel with Doppler effect: Sum rate performance measurement

a) In case users K=4, N=4:

Figure 3.5 - 3.6 show the average throughput versus SNR for the case $N = 4$ and $K = 4$ with different Doppler effects. Figure 3.5 shows the performance for receiver's moving 100km/hr using a carrier frequency of 2GHz. The symbol duration is considered as $T = (1 \times 10^{-3})/14\ s$ since in LTE each subframe lasts 1ms and contains 14 OFDM symbols and the sampling rate is $T_s = 4.4 \times 10^{-6} s$ with an oversampling factor $Q = 16$. Figure 3.6 shows the performance for the speed 200k/hr. Simulation results show the fact that Doppler effect degrades performance compare to channel with no effect. For the speed 100km/hr and 200km/hr, the performance degrades almost 11% and 15% compare to the performance with no speed.

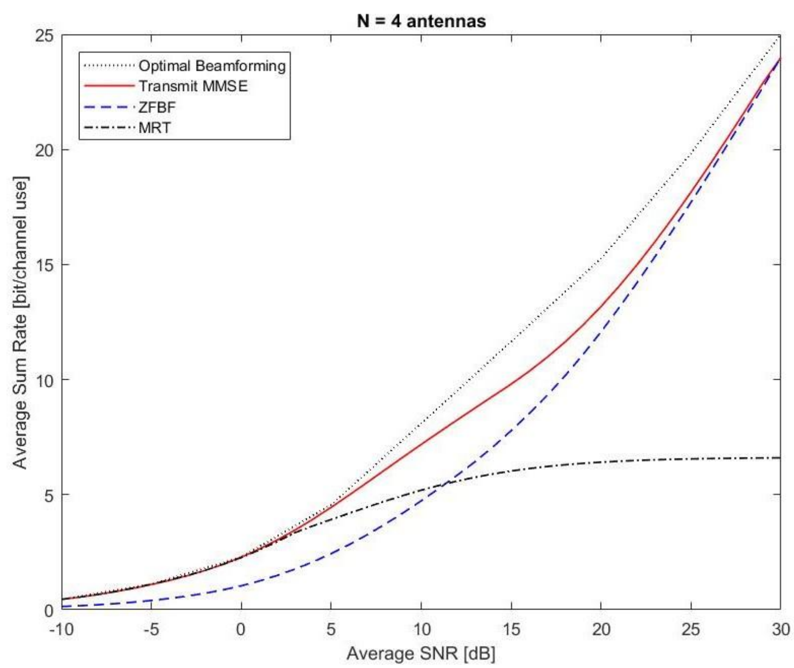


Figure 3.5: Average sum throughput vs. SNR with velocity, $v = 100$ km/hr

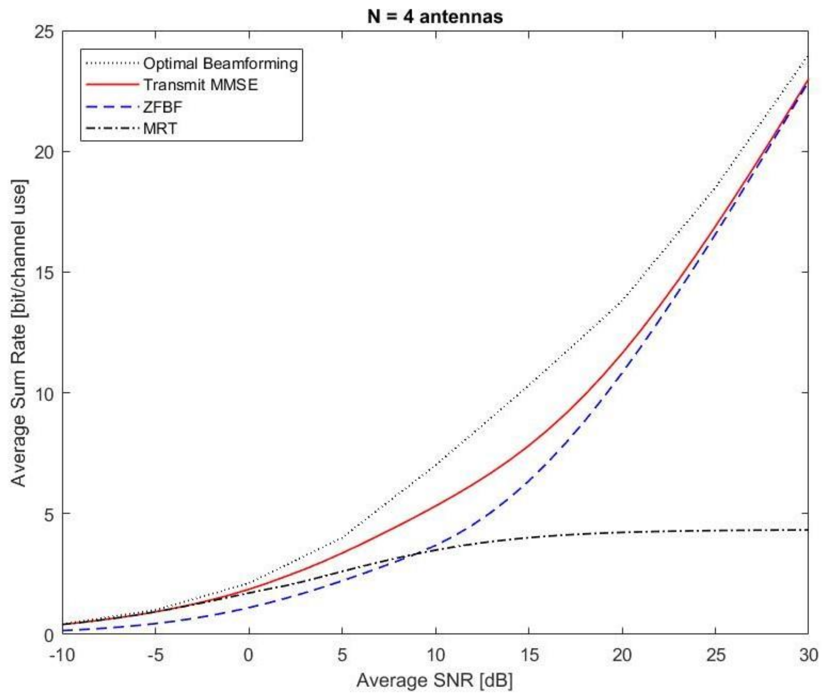


Figure 3.6: Average sum throughput vs. SNR with velocity, $v = 200$ km/hr

b) In case users K=4, N=12

Figure 3.7 – 3.8 show the average throughput versus SNR for the case $N = 12$ and $K = 4$ with different Doppler effects. Simulation results in Figure 3.7 and Figure 3.8 correspond to cases where we increase the speed to the 100km/hr and 200km/hr. They show the variation of the performance degradation that is almost the same and which is 13% when compared to the performance when the mobile is not moving.

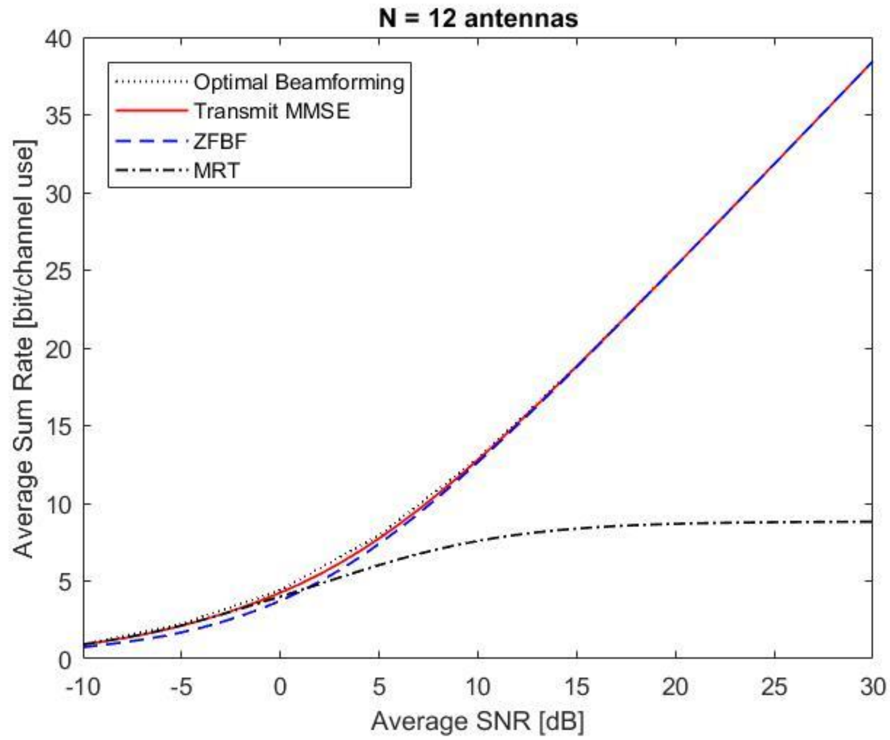


Figure 3.7: Average sum throughput vs. SNR with velocity, $v = 100$ km/hr

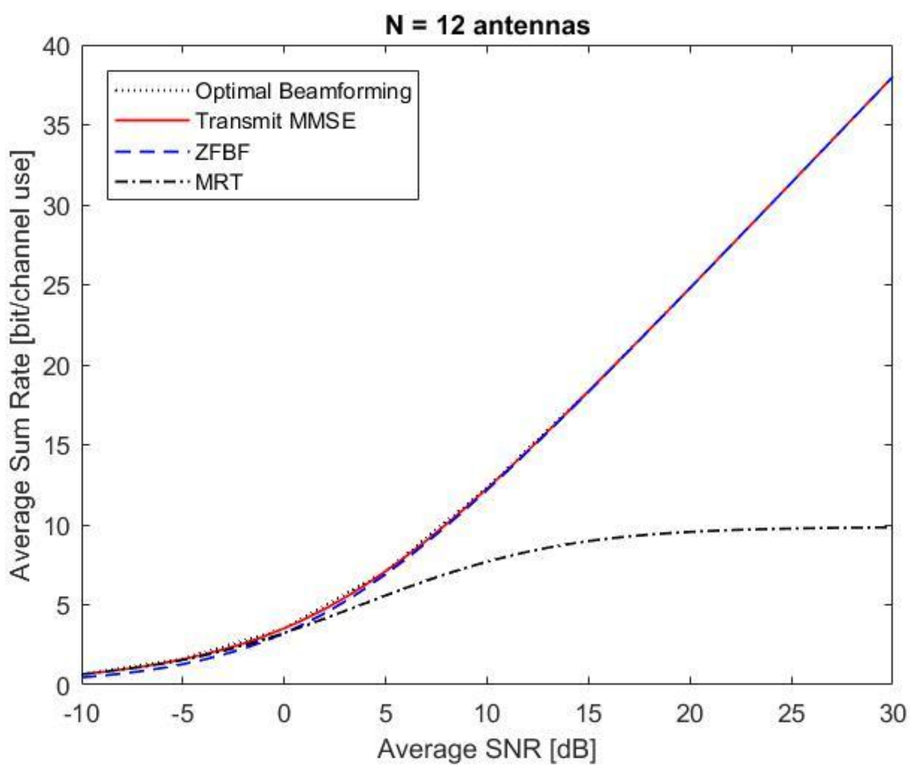


Figure 3.8: Average sum throughput vs. SNR with velocity, $v = 200$ km/hr

To conclude this chapter, when the number of antennas is equal to the number of users, that is when $K = N$, the Doppler effect or multipath delays effect significantly reduce the performance compared to the case $K \ll N$. Moreover, a large number of antennas reduces the necessity of optimal beamforming but it produces a huge implementation complexity with the number of users K.

CHAPTER 4 PROPOSED ALGORITHMS FOR TRANSMIT ANTENNAS SELECTION AND OPTIMAL BEAMFORMING

4.1 An Iterative Solution for Transmit Antennas Selection and Optimal Beamforming

The more antennas at the transmitter or the receiver are equipped with, the better the data rate/link reliability. However, massive MIMO implies challenges such as hardware impairments and signal processing complexity which may limit the number of antennas in practice. Thus, it is interesting to analyze MIMO networks in the presence of large but finite number of antennas. Particularly, several antenna selection algorithms are presented in which only a set of antennas are activated based on the channel quality, transmit power to utilize the diversity of large MIMO systems. In the following section we will consider antenna selection scheme and investigate the performance of MMSE, ZFBF and MRT beamforming schemes.

Many works have done on antennas selection strategies for massive MIMO. For example, joint antenna selection and power minimization problem in [22], [31], antennas selection for a continuous and burst communication [23]. Literature studies shows that the antenna selection is a NP hard problem. To solve the NP hard problem, [31] proposed heuristic searching algorithms where the complexity increase with the number of total antennas, N. Similarly, [22], [31] proposed l_1, $l_{1/2}$ norm to obtain a sparse solution of the antennas selection problem. However, all of these proposed schemes have computational complexity to select the best set of antennas as the total subsets which is 2^N increased with the value N.

We propose a simple framework to select the number of antennas L among the total number of antennas N instead of searching a best set of antennas among all subsets as proposed in earlier work.

4.1.1 Basic Model and Problem Formulation

Assuming the same system model defined in chapter three that is a single base station with multiple antennas N and K single antenna receiver. The K different data signals are separated spatially using the linear beamforming vectors $\boldsymbol{w}_1, \dots \dots \dots, \boldsymbol{w}_K \in \mathbb{C}^{N\times 1}$ and the transmission power $\|\boldsymbol{w}_k\|^2$ is allocated to the user k. The received signal at user k is given by,

$$y_k = \boldsymbol{h}_k^H \boldsymbol{w}_k + \sum_{k' \neq k} \boldsymbol{h}_k^H \boldsymbol{w}_{k'} + n_k \tag{4.1}$$

where $\boldsymbol{h}_k \in \mathbb{C}^{N\times 1}$ is the channel vector from the base station to user k and $z_k \sim \mathcal{CN}(0, \sigma^2)$ is the additive receiver noise. The SINR at user k^{th} given as

$$SINR_k = \frac{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{k' \neq k} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + \sigma^2} \tag{4.2}$$

The design problem is to minimize the total transmit power, subject to prescribed receive SINR thresholds γ_k at each user that is,

$$\begin{aligned} \min_{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K} \quad & \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \\ subject\ to \quad & SINR_k \geq \gamma_k \end{aligned} \tag{4.3}$$

Problem (4.3) can be reformulated as a convex, second order cone programming problem [15] and thus can be solved efficiently. Since the phases of $\boldsymbol{w}_k$ will not change the objective function and constraints of (4.3), the SINR constraints are equivalent to the following second order cone constraints:

$$\frac{1}{\sqrt{\gamma_k}} \Re(\boldsymbol{h}_k^H \boldsymbol{w}_k) \geq \sqrt{\sum_{k' \neq k} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + \sigma^2} \tag{4.4}$$

4.1.2 Antennas Selection

Suppose that only $L \leq N$ antennas need to be selected and thus, only L antennas can be transmitting simultaneously. The goal is to jointly select the L out of N antennas and find the corresponding beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^{K}$ that satisfied the SINR constraints in problem (4.3). Let us define a $N \times 1$ vector, $\boldsymbol{n}$, whose elements are either 0 or 1. The n^{th} element of $\boldsymbol{n}$, $\boldsymbol{n}_n$, is the antenna selection coefficient for the n^{th} antenna. Hence, $\boldsymbol{n}_n = 1$ if the n^{th} antenna is selected, it is zero otherwise. The joint problem can be written as,

$$\min_{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K} \quad \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2$$

$$subject\ to \quad \frac{1}{\sqrt{\gamma_k}} \Re(\boldsymbol{h}_k^H \boldsymbol{w}_k) \geq \sqrt{\sum_{k' \neq k} |\boldsymbol{h}_k^H \boldsymbol{w}_{k'}|^2 + \sigma^2}$$

$$\sum_{n=1}^{N} \boldsymbol{n}_n = L \tag{4.5}$$

Problems in (4.5) can be infeasible due to the strong interference, strict SINR constraints or insufficient number of antennas. Thus, the number of antennas need to be selected that is L is dependent on the number of users, channel coefficient and SINR constraints.

4.1.3 Sparse Beamforming Framework

In this section, we propose an iterative algorithm to estimate the minimum number of antennas L that satisfied the QoS requirements of each user. The main motivation is to induce sparsity in the beamformer which corresponds to switching off the antennas at base station.

Figure 4.1 shows a three stages frame work to use minimum number of antennas, L from the antennas *N*. Specifically, in the first stage, we propose an ordering rule to determine the priority for the sets of antennas that should be switched off. In the second stage, for the given number of antennas $L_{initial}$, the SINR constraints defined in (4.3) are checked. If the QoS requirements are unattainable (due to inter-user interference), the number of selected antennas $L_{initial}$ is increased otherwise the number of antennas $L_{initial}$ is decreased iteratively in order to estimate the minimum number of antennas L.

1) Sets of antennas ordering: We will first give the priorities to the sets that have the smallest number of elements. The number of elements in the sets is increased exponentially as given below,

$$\mathcal{L}_1 < \mathcal{L}_2 < \cdots < \mathcal{L}_M, \quad M = \log_2 N$$

$$L_{intial} = |\mathcal{L}_m| = 2^m, \quad 1 \leq m \leq M \tag{4.6}$$

The antenna elements are randomly selected at each set.

2) Minimizing the transmission power with SINR constraints: After selecting the initial number of antennas, L_{intial}, the corresponding beamforming vector for the selected antennas can be

given by any heuristic beamforming . For example, MMSE beamforming directions can be given as,

$$\boldsymbol{h}_{kL} = \boldsymbol{h}_k \times \boldsymbol{n}$$

$$\overline{\boldsymbol{w}}_{kL} = \left(\sum_{k=1}^{K} \boldsymbol{h}_{kL}\boldsymbol{h}_{kL}^H + \sigma^2 \boldsymbol{I}\right)^{-1} \boldsymbol{h}_{kL} \tag{4.7}$$

After fixing the beamforming directions the power allocations can be formulated by so called waterfilling solutions as given in (3.25). For the given set of antennas L, the SINR become

$$SINR_k = \frac{p_k |\boldsymbol{h}_{kL}^H \overline{\boldsymbol{w}}_{kL}|^2}{\sigma^2 + \sum_{k' \neq k} p_{k'} |\boldsymbol{h}_{kL}^H \overline{\boldsymbol{w}}_{k'L}|^2}$$

The SOCP problem formulated in (4.3) is solved by CVX. If the SINR requirements are unattainable for the given number of antennas, then the first stage is repeated again after increasing the order number.

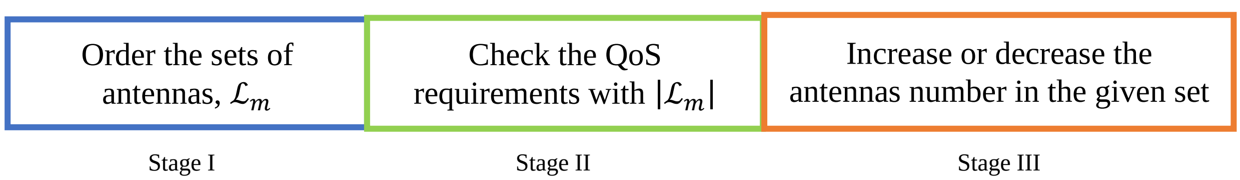


Figure 4.1: Frame work to use limited antennas from massive antennas

3) Estimate the minimum number of antennas L: If the problem (4.3) is feasible for the given number of $L_{initial}$, then reduce the number of antenna elements by M and check the QoS requirements. Repeat the stage three as long as the problem is feasible and finally determine the minimum number of antennas L.

$$L = L_{initial} - M$$

The number of antennas is reduced by switching off randomly in the antenna element vector.

Algorithm 1: Estimate the number of antennas that satisfy QoS requirements

1. Order the number of antennas as $\mathcal{L}_1 < \mathcal{L}_2 < \cdots < \mathcal{L}_M$
2. For the given number of antennas $L_{initial} = |\mathcal{L}_m|$, Check the QoS requirements in (4.3)
   - **a)** **If** the problem (4.3) feasible then **go to step 3**
   - **b)** **Else** $m = m + 1$ and **go to step 1**
3. **If** the problem (4.3) is feasible, **then** $L_x \leftarrow L_{initial}$
4. $L_{x_i} = L_x - M$ at i^{th} iteration
   - **a)** For the given L_{x_i}, check again the feasibility in (4.3)
     - i. **If** it is feasible, **then**, $L_x \leftarrow L_{x_i}$
     - ii. i = i+1 and **go to (a)**
     - iii. **else,** $L = L_{x_{i-1}}$
5. **End** and results the selected minimum number of antennas, L

In the following section, we study the performance of the ZFBF, MMSE and MRT beamforming schemes with antenna selection procedure defined in Algorithm 1.

4.1.4 Performance Studies of ZFBF, MMSE, MRT

In this section, we consider a fading channel with moving receivers, v= 150km/hr which implies the Doppler spread $f_d = 280.063$ Hz. The total number of antennas is, $N = 16$. The antenna elements are randomly chosen. We also defined the value of $M = 2$. Figure 4.2 shows approximately minimum number of antennas is required for different number of users. The performance is analyzed for SNR = 20dB and the SINR threshold γ_k for each user's is randomly defined between the range $0\ to\ 7dB$. Figure 4.3 shows the average sum rate throughput for different values of L and compares the performance with a fading channel without Doppler effects. Note that for a channel affected with motion and delays, required more antennas compare to a static channel. Algorithm 1 does not guarantee the exact minimum number of antennas to satisfy the QoS constraints. However, it gives a reasonable optimum number of antennas with only few iterations.

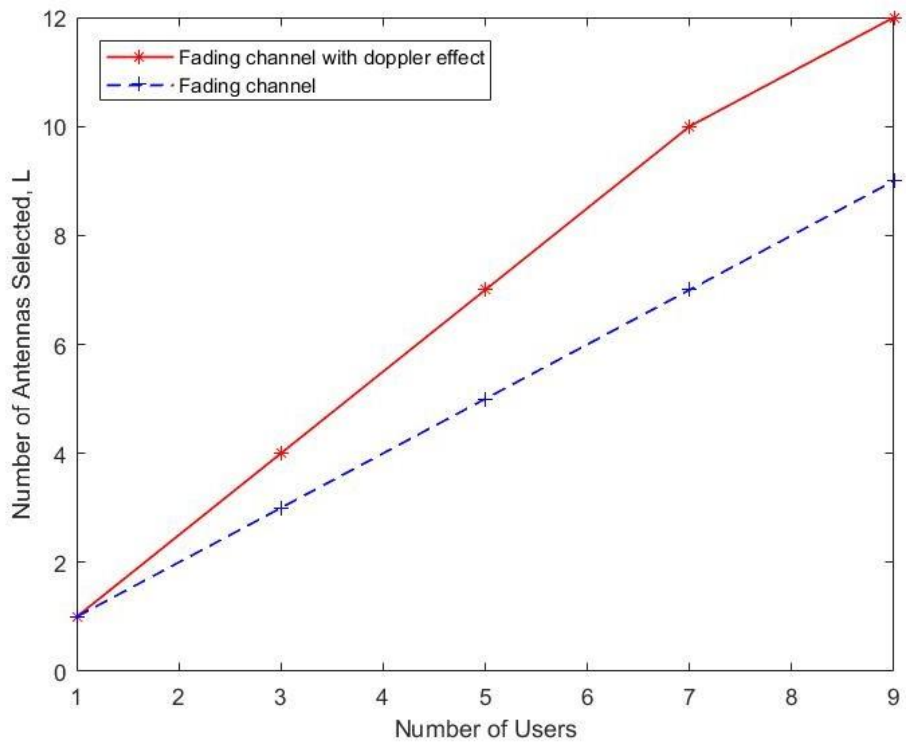


Figure 4.2: Number of antennas selected, L vs. Number of users, SNR = 20dB

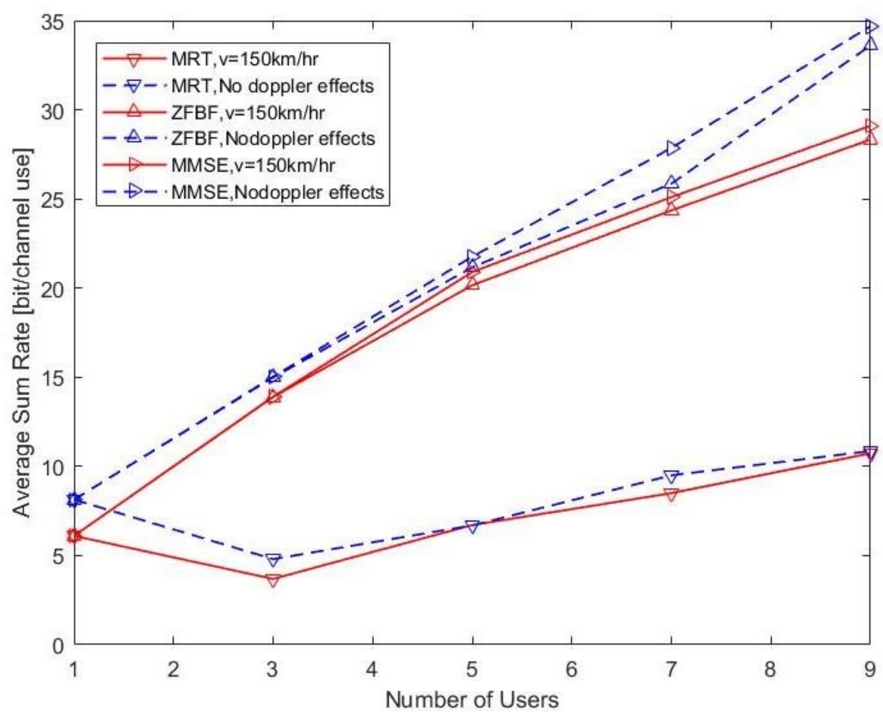


Figure 4.3: Average sum rate vs. Number of users, SNR = 20dB

Figure 4.4 shows a comparison of sum throughput between best set of antennas selection and a set selection where the antennas are randomly selected. To select the best set, we use maximum throughput as objective function and the best set is exhaustively searched. In Figure 4.4, the value of L is predefined as we found from the Figure 4.3, the value of L is dependent on the number of users and the channel quality. Result shows that best set selection can improve the performance only when the value of L is small at a cost of huge computational complexity.

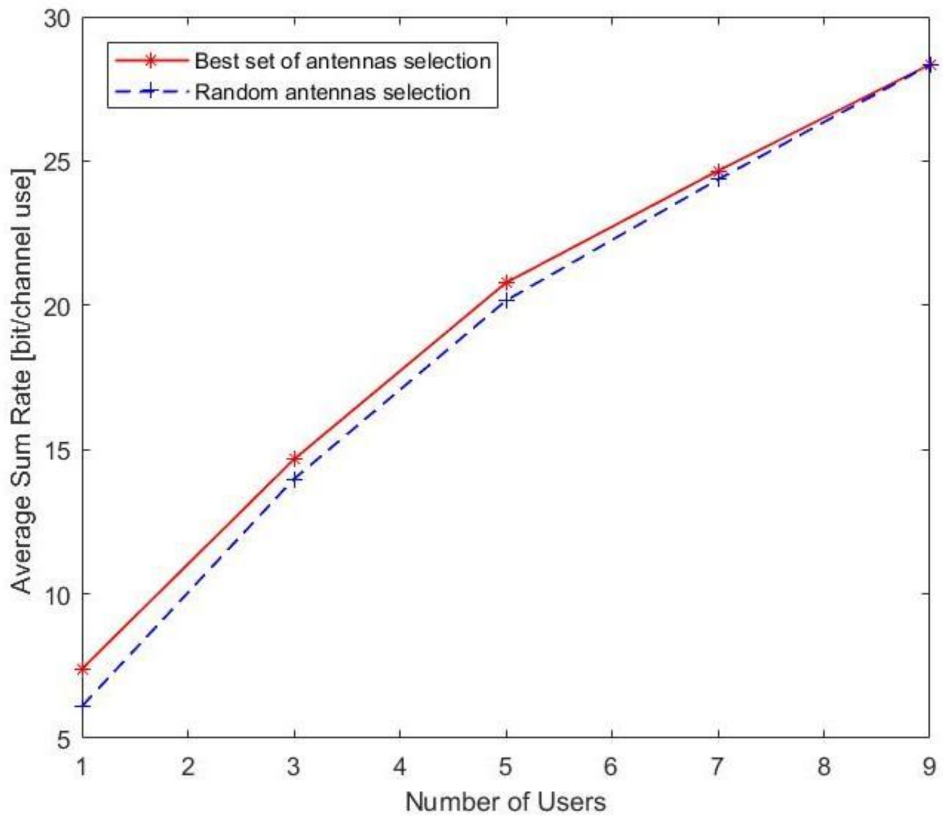


Figure 4.4: Average sum rate vs. Number of users, SNR = 20dB

4.2 A Genetic Algorithm Approach for Optimal Multi-User Transmit Beamforming

Chapter three shows a simple structure of optimal beamforming and characterize some of its properties. This structure provides a theoretical foundation for practical low complexity beamforming schemes. Based on the explained beamforming structure, we address the problem of downlink beamforming in a power-controlled network. The total transmitted power is limited and the channel information is available at the transmitter. The goal is to maximize the sum throughput function $f(SINR_1, \dots \dots SINR_K) = \sum_{k=1}^{K} log_2(1 + SINR_k)$ while the total transmit power is limited by P. We propose in this chapter an efficient scheme using genetic algorithm (GA) for optimal transmit beamforming of multiple users. The proposed genetic algorithm is generic in the sense that it can be implemented for different objective functions and different channel models. Simulation results are compared to published results obtained from the BRB optimization technique. Our results show that the proposed GA based algorithm reaches almost the same throughput as the BRB based optimal approach with less implementation complexity.

4.2.1 System Model and Problem Statement

The system model and problem statements are extensively explained in chapter three. Here, we briefly describe the problem statement. Consider a downlink scenario where K users must achieve individual target SINRs for successful communication. The total transmission power is limited by P. Each user has its own objective $r_k = g_k(SINR_k) = \log_2(1 + SINR_k)$ to be optimized, thus there are K different objectives that typically are conflicting. Without loss of generality, our resource allocation problem is formulated as,

$$\underset{\boldsymbol{w}_1,\dots\dots,\boldsymbol{w}_K}{\text{maximize}}\, f(g_1(SINR_1), \dots \dots., g_K(SINR_K))$$

$$subject\ to \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \le P \tag{4.8}$$

Since the performances of different users are coupled by both power constraints and inter-user's interference, there is generally not a single transmit strategy that maximize the performance for all users. For example, $SINR_k$ in (3.2) improves if less interference is caused to user, k.

However, decreasing the interference at k typically requires decreasing the useful signal power at the other users and thus degrading their SINRs. Therefore, we need to find a fine tuning of user's individual utility function represented as $r_1, r_2, \dots \dots\ r_K$ that satisfies the power constraints and maximize the utility function in (4.8). Previous work in [15],[3] formulates the problem and shows that iteratively solving another optimization problem that is the total transmission power minimization with SINR constraints is equivalent to solving the problem defined in (4.8)

$$\begin{aligned} &\underset{\boldsymbol{w}_1,\dots,\boldsymbol{w}_K}{\text{minimize}} \quad \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \\ &subject\ to\ SINR_k \geq \gamma_k \end{aligned} \tag{4.9}$$

The problem defined in (4.9) is relatively easier to solve than the problem defined in (4.8). The beamforming vectors that solve the problem (4.9) must satisfy the total power constraints in (4.8) [3], [15] as (4.9) gives the beamforming that achieves the given $SINRs$ using the minimal amount of power. Consider the set of all feasible operating points $\boldsymbol{r} = [r_1 \dots \dots \dots r_K]$ which we call the performance region. The achievable performance region $\mathcal{R} \subseteq [\boldsymbol{0}, \boldsymbol{u}]$ is

$$\mathcal{R} = \left\{ \left(g_1(SINR_1), \dots \dots, g_K(SINR_K) \right) : (\boldsymbol{w}_1, \dots \dots, \boldsymbol{w}_K) \in \mathbb{W} \right\} \tag{4.10}$$

Where $\mathbb{W}$ is the set of feasible transmit strategies:

$$\mathbb{W} = \left\{ (\boldsymbol{w}_1, \dots \dots, \boldsymbol{w}_K) : \boldsymbol{w}_k \geq 0, \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \leq P \right\} \tag{4.11}$$

The utopia point $\boldsymbol{u}$ is the unique solution to (4.8) when all users would get the same rate as if they were the only user in the system. In the following section, we propose a metaheuristic scheme for searching a transmit strategy $\boldsymbol{w}_1, \dots \dots, \boldsymbol{w}_k$ that satisfies the power constraints and maximizes the sum throughput of the system.

4.2.2 Proposed Genetic Algorithm Description

Genetic algorithms are a class of search technique that use the mechanics of natural selection and genetics to conduct a global search of a solution space. The goal of the search is to find a good solution to the given problem. Other optimization techniques such as gradient descent search a region of the solution space around an initial guess for the best local solution. For problems that have a small number of parameters and, hence, a small solution space, gradient descent methods perform very well because they are able to search a significant portion of the entire solution space. However, as the number of parameters and, hence, the size of the solution space increases, the quality of the solution depends upon the location of the initial guess. If the initial guess falls in a region of the solution space where all the local solutions are poor, a local search is limited to finding the best of these poor solutions.

Binary genetic algorithms (GAs) have been proposed in massive MIMO systems for antennas selection [23], to obtain the position and orientation of a MIMO array [38], to optimize the element spacing and lengths of Yagi-Uda antennas [49]. We have proposed the GA algorithm to solve a problem where the values of the variables are continuous. Thus, the size of the chromosome is very large as the variables are represented by floating point numbers.

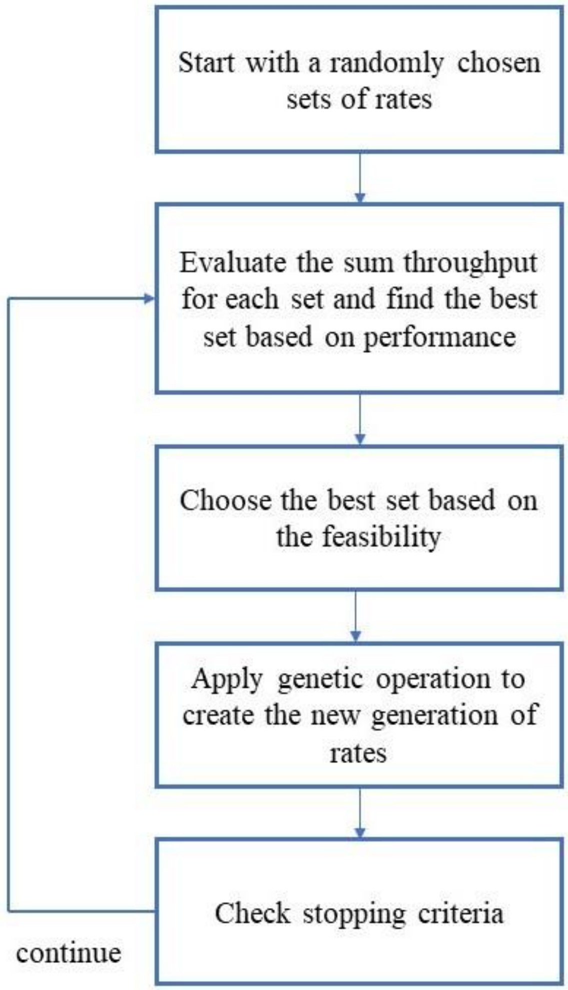


Figure 4.5: Flow diagram for the genetic algorithm used in this work

GA is constructed from a number of distinct components. The main components are the population initialization, the fitness function, selection, recombination and the evolution scheme.

4.2.2.1 Chromosome Formulation (or Search Space) and Initial Population:

Firstly, the search space is defined by a matrix and the space is bounded by the utopia point of each user. Proposed algorithm starts with a matrix $\boldsymbol{R}^{G\times K} \subseteq [\mathbf{0}, \boldsymbol{u}]$ where each column is bounded by a lower and upper bound. The maximum number of rows defined the maximum number of population. The original points are 0 for all users and upper bound,

$\boldsymbol{u} = [u_1, u_2 \ldots \ldots u_K]$ where u_k is the rate obtained by MRT as each user would get the rate if it was the only user in the system. Thus,

$$\boldsymbol{R} = \begin{bmatrix} r_{1_1} & \cdots & r_{1_K} \\ \vdots & \ddots & \vdots \\ r_{G_1} & \cdots & r_{G_K} \end{bmatrix} \tag{4.12}$$

Here G_k is the maximum number of chromosomes is created to start the algorithm. In other words, the matrix $\boldsymbol{R}$ is the search space to start our proposed GA.

After a suitable representation of the search space which contains possible rates of each user, we initialize S possible sets of rates $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots \ldots . . \boldsymbol{r}_S\}$, where $\boldsymbol{r}_1 = [r_1, r_2, \ldots . r_K]$ is the vector of user's rate. This initial population is usually created randomly and defined by a matrix $\breve{R}^{S \times K}$

4.2.2.2 Fitness Function and Selection:

Next, for each possible sets of rates $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots \ldots . . \boldsymbol{r}_S\}$, we evaluate the sum throughput $f\big(g_1(SINR_1), \ldots \ldots, g_K\,(SINR_K)\big) = \sum_{k=1}^{K} g_k(SINR_k)$. The set that results maximum throughput referred to as the best set $\boldsymbol{r}_{best}$ at each iteration. Now to check the power constraints in (4.8), we solve the problem formulated in (4.9). The problem is reformulated in (3.4) which is a second order cone program (SOCP). The only difference in (4.8) and (4.9) is that the $SINRs$ are predefined in (4.9). Thus, for the given best set of rates, the required $SINRs$ are,

$$\gamma_k = SINR_k^* = \; g_k{}^{-1}\,(r_{best}) \tag{4.13}$$

By injecting the value of γ_k in equation (4.9) as SINR constraints, we check the feasibility of the problem. For the given rate, if the problem is feasible then the selected best set referred as the queen set $\boldsymbol{r}_{queen}$. Otherwise, the local feasible set that is MMSE rates is defined as queen set.

4.2.2.3 Recombination and Evaluation of New Generation:

We keep the queen set for producing the next generations in the iteration process. Here, J is the number of sets to be produced from the queen set. In other words, number of new generations from the queen set. This is achieved by applying small modifications to the queen set that is by increasing the rate within a range. Thus, after each iteration, the selected queen leads to the best optimum set of rates. The rest set of rates are selected randomly from the matrix $\boldsymbol{R}$ and the iterations continue until the stopping criteria meet. In summary, the new generation for the next iteration is produced in two ways:

1) Here, J is the number of new generations from the queen set

$$\boldsymbol{r}_j = \boldsymbol{r}_{queen} + \varepsilon_j, \qquad a_0 \leq \varepsilon_j \leq b_0 \quad and \;\; J < S \tag{4.14}$$

2) The rest of the sets or rows $(S - J - 1)$ of $\breve{\boldsymbol{R}}$ are randomly chosen from the matrix $\boldsymbol{R}$

4.2.2.4 Termination Conditions for the GA

In generally, GA has two types of stopping criteria. First, maximum number of iterations: when the generation reaches to this predefined value, it stops and provides the best solution. Secondly, the convergence criteria. In this work, we study the performance with maximum number of iterations.

Algorithm 2: Genetic Algorithm (GA) for Optimum Beamforming

Result: The best feasible solution $\boldsymbol{r}_{best\_feasible}$ found by the algorithm

Input: Initial local feasible solution, $\boldsymbol{r}_{local\_feasible}$, total power P, upper bound, $\boldsymbol{u}$

Input: Initial matrix $\boldsymbol{R}^{G\times K}$such that $\boldsymbol{R} \subseteq [\boldsymbol{0}, \boldsymbol{u}]$

1. Consider S possible sets of rates $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \dots \dots . . \boldsymbol{r}_S\}$ which are subsets of the matrix $\boldsymbol{R}^{G\times K}$ and sets are randomly selected as $\breve{\boldsymbol{R}}^{S\times K} \subseteq \boldsymbol{R}^{G\times K}$
2. For each strategy $s = 1,2 \dots \dots S$ and i^{th} iteration evaluate the instantaneous value of the objective function $f_s(r_1, \dots . . r_K) = \sum_{k=1}^{K} \log_2(1 + SINR_k)$ that is the throughput given by equation (4.8)
3. Find the set of rates which results in the best value of the objective function (the queen) that is $\boldsymbol{r}_{queen}$ where $f_s(r_1, \dots . . . , r_K) \leq f_i(r_1, \dots . . . , r_K)$
4. Check feasibility of $\boldsymbol{r}_{queen}$ by solving the SOCP problem in (3.4)
   if *infeasible* then
   set $\boldsymbol{r}_{best\_feasible} = \boldsymbol{r}_{local\_feasible}$
   else
   set $\boldsymbol{r}_{best\_feasible} = \boldsymbol{r}_{queen}$
5. $\breve{\boldsymbol{R}}_1 \leftarrow \boldsymbol{r}_{queen}$
6. Generate $J \ll S$ sets of rates $\boldsymbol{r}_j^{new}, j = 1,2 \dots J$ around $\boldsymbol{r}_{queen}$. These sets are generated by small changes in the queen by increasing the rates by a small amount $\boldsymbol{r}_{queen} + \varepsilon_j$
   $\boldsymbol{r}_j^{new} \leftarrow \boldsymbol{r}_{queen} + \varepsilon_j$, $\forall\ \varepsilon_j \in (a_0, b_0)$ where $a_0 = 0.01, b_0 = 0.08$
7. $\breve{\boldsymbol{R}}_{j+1} \leftarrow \boldsymbol{r}_j^{new}, j = 1,2 \dots J$
8. Generate the remaining sets $j = J + 2, \dots . S$ randomly with the same procedure in step 1.
9. **Go to step 2** and continue for $N_{\max\_iterations}$ where $N_{\max\_iterations}$ is the number of iterations.

4.2.3 Performance Analysis With GA Based Optimum Beamforming Scheme:

For the simulation results, we consider two types of fading channel. Figure 4.6 shows the result for a simple Rayleigh fading channel with no multipath effects. We have considered $N = 4$ antennas and the number of users $K = 4$. For the proposed GA, the number of total chromosomes, $G = 500$, $S = 8$ and $J = 5$. The value of ε_j is defined based between the range 0.01 to 0.08. Figure 4.7 shows the sum throughput for a Rayleigh fading channel with Doppler effects. For the Doppler effects the speed is considered 150km/hr. As seen from Figure 4.6-4.7, proposed scheme leads to the near optimum value obtained from the BRB algorithm with very few iterations $N_{\max\_terations} = 80$.

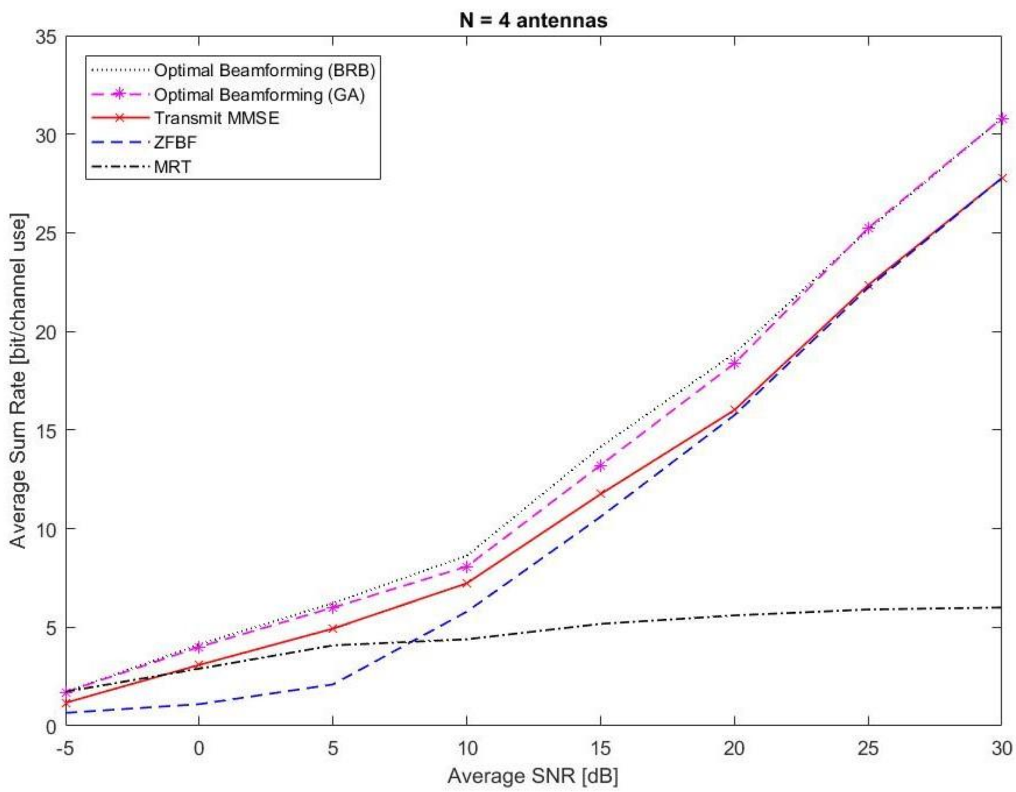


Figure 4.6: Sum throughput vs. SNR

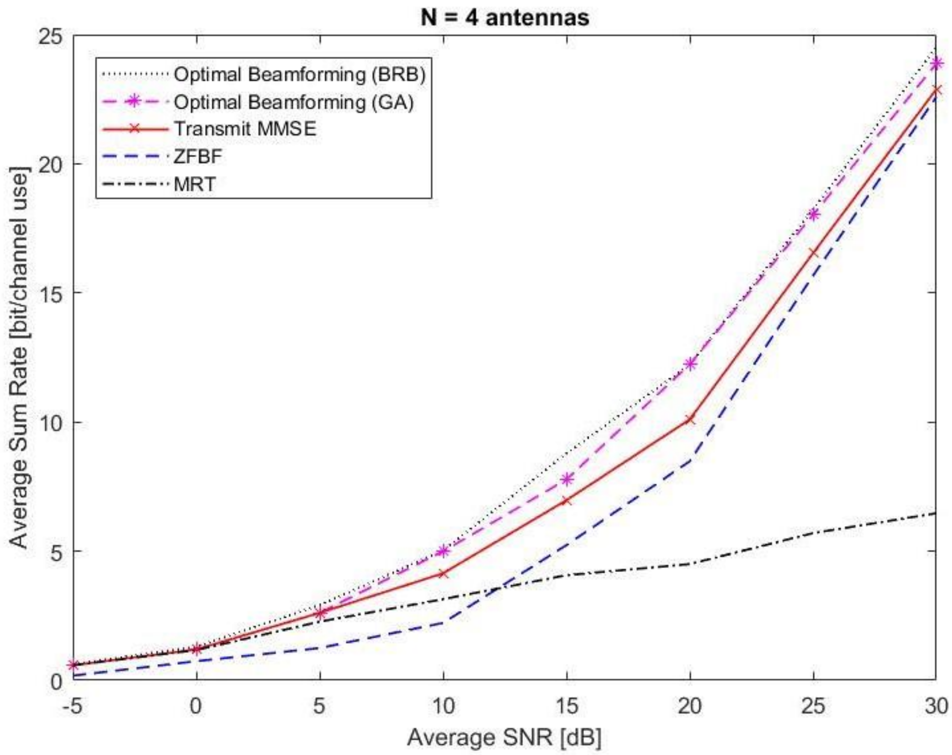


Figure 4.7: Sum throughput vs. SNR with v=150km/hr

Figure 4.8 shows the number of iterations required for the proposed GA algorithm to reach the 100% same performance as the BRB algorithm. Here, we consider the same parameters described for Figure 4.6. We plot the relative achievable throughput $\Delta = \frac{\mu_{N_{\max\_iterations}}}{\mu_{BRB}} \%$, where $\mu_{N_{\max\_iterations}}$ is the throughput achieved with $N_{\max\_terations}$ iterations while μ_{BRB} is the throughput achieved with BRB algorithm.

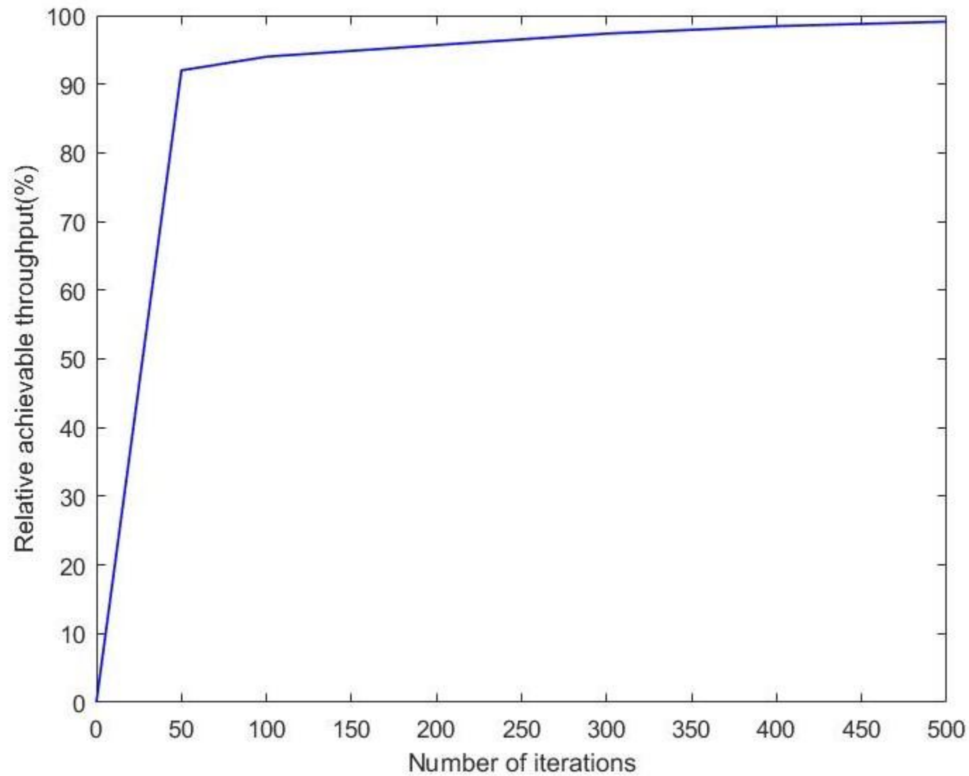


Figure 4.8: An example of the convergence process of the proposed GA based algorithm

4.2.4 Implementation Complexity of the Proposed Algorithm

Implementation complexity of the proposed GA algorithm depends on few parameters such as number of users K , number of iterations $N_{\max\_terations}$ and initial number of sets S. The major complexity is to solve the SOCP problem for each set S where the complexity to solve the SOCP problem depends on number of users K. If we define the complexity of SOCP as $O_{socp}(K)$, then the implementation complexity can be given by $N_{max}K(O_{socp}(K)\ )$.

Few facts can be noticed for the proposed genetic algorithm:

- The proposed algorithm leads to almost same results as the BRB algorithm with fewer iterations as showed in Figure 4.8. Proposed GA reached to almost 92% of the BRB algorithm with only 100 iterations. Therefore, the algorithm is reasonably fast and can be easily implemented.
- The algorithm considers no constraints on number of users K.
- The proposed scheme results approach the best optimal solution when the number of iterations increases asymptotically.

CHAPTER 5 CONCLUSION

In this thesis, we have studied a simple solution structure of an optimal multiuser beamforming problem. We have analyzed the performance of MRT, ZFBF, MMSE and optimal beamforming in a proper fading channel. Simulation results showed that the properties of all these beamforming schemes do not change the performance curve. However, the effects of Doppler spreading degrade the performance by around 10-15% compared to the case when channel is assumed to be static. The simulation results also characterize the performance degradation for two cases: the first case is when the number of antennas is equal to the number of users, the degradation is greater when the number of antennas is much larger than the number of users. Moreover, the performance gaps between the conventional beamforming that is MRT, ZFBF, MMSE and optimal beamforming is significantly noticeable in the first case while all the beamforming schemes performs almost the same except the MRT in the second case.

An important fact has received much attention when we consider multiple antennas at the base station: the use of large arrays of antennas in the performance analysis. When there are more antennas at the transmitter better data rates are obtained. However, large MIMO implies signal processing complexity and hardware impairments which may limit to use the large arrays of antennas in practical scenarios. Previous works have done to select the best set of antennas by solving various optimization problems such as total power minimization, maximization of the sum throughput. Selecting the best set of antennas is a NP hard problem which increases the implementation complexity as the size of the antennas arrays becomes large. In this work, we propose a simple scheme to minimize the set of antennas from large arrays of antennas. Instead of blindly searching which set is the best from all subsets, we simply select a set of antennas that adequately satisfies the QoS requirements. We proposed an iterative solution to randomly select the minimum number of antennas that is required to satisfy the QoS requirements of each user. The proposed method reduced the number of antennas that is unnecessary when there is small number of users. Thus, it also reduced the hardware complexity and signal processing complexity in the case where large arrays of antennas are used at the base station. The proposed iterative solution was implemented in two types of channels: static channels and channels with Doppler fading and results confirmed that the static channel required fewer antennas than then time varying channels which are expected. Our proposed method is very simple in the sense that it has low implementation complexity to select a suitable set of antennas, while at the same time it reached to a minimum number of antennas that allow satisfying the QoS constraints.

Another major contribution of this thesis is to propose an efficient heuristic searching algorithm for optimal beamforming with low implementation complexity. We have solved the optimal beamforming problem using a genetic algorithm which is very general and easy to implement. Simulation results showed that the proposed algorithm performs almost the same as the branch reduce bound algorithm. The genetic algorithm reached 92% of the performance obtained from the BRB algorithm with only few iterations. Simulation results also showed that 100% can be achieved by increasing the number of iterations which is the termination criteria for GA. In both algorithms, the complex SOCP problem needs to be solved in each iteration. However, in the BRB algorithm, the feasibility need to checked twice compared to the GA where it is validated in each iteration.

REFERENCES

[1] F. Rusek, D. Persson, B. Lau, E. Larsson, T. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 40–60, 2013.

[2] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms," IEEE Trans. Signal Process., vol. 59, no. 3, pp. 1142–1157, 2011.

[3] Emil Björnson, Mats Bengtsson, and Björn Ottersten "Optimal Multiuser Transmit Beamforming: A Difficult Problem with a Simple Solution Structure" IEEE Signal Processing Magazine, July 2014

[4] M. Joham, W. Utschick, and J. Nossek, "Linear transmit processing in MIMO communications systems," IEEE Trans. Signal Process., vol. 53, no. 8, pp. 2700–2712, 2005.

[5] M. Sadek, A. Tarighat, and A. Sayed, "A leakage-based precoding scheme for downlink multi user MIMO channels," IEEE Trans. Wireless Commun., vol. 6, no. 5, pp. 1711–1721, 2007.

[6] F. Rusek, D. Persson, B. Lau, E. Larsson, T. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," IEEE Signal Process. Mag., vol. 30, no. 1, pp. 40–60, 2013

[7] T. Lo, "Maximum ratio transmission," *IEEE Transactions on Communications*,vol. 47, no. 10, pp. 1458‑1461, 1999.

[8] M. Joham, W. Utschick, and J. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2700‑2712, 2005.

[9] H. Liu and G. Xu, "Multiuser blind channel estimation and spatial channel pre-equalization, " in *Proceedings of IEEE International Conference on Acoustics,Speech, and Signal Processing*, pp. 1756‑1759, 1995.

[10] G. Montalbano, I. Ghauri, and D. Slock, “Multiuser blind channel estimation and spatial channel pre-equalization,” in *Proceedings of Asilomer Conference on Signals, Systems, and Computers*, pp. 1337–1341, 1998.

[11] M. Sadek, A. Tarighat, and A. Sayed, “A leakage-based precoding scheme for downlink multi-user MIMO channels,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 1711–1721, 2007.

[12] A. Tarighat, M. Sadek, and A. Sayed, “A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to-leakage ratios,” in *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, pp. 1129–1132, 2005.

[13] Hyoungjoo Lee, Illsoo Sohn, Donghyun Kim, and Kwang Bok Lee “Generalized MMSE Beamforming for Downlink MIMO Systems” IEEE ICC 2011 proceedings.

[14] Martin Schubert and Holger Boche, “Solution of the Multiuser Downlink Beamforming

Problem With Individual SINR Constraints” IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, VOL. 53, NO. 1, JANUARY 2004

[15] Emil Bj¨ornson and Eduard Jorswieck, “Optimal Resource Allocation in Coordinated Multi-Cell Systems” Foundations and Trends in Communications and Information Theory Vol. 9, Nos. 2–3 (2012) 113–381

[16] E. Bj¨ornson, G. Zheng, M. Bengtsson, and B. Ottersten, “Robust monotonic optimization framework for multicell MISO systems,” *IEEE Transactions on Signal Processing*, vol. 60, no. 5, pp. 2508–2523, 2012.

[17] H. Tuy, F. Al-Khayyal, and P. Thach, “Monotonic optimization: Branch and cut methods,” in *Essays and Surveys in Global Optimization*, (C. Audet, P. Hansen, and G. Savard, eds.), Springer US, 2005.

[18] S. Joshi, P. Weeraddana, M. Codreanu, and M. Latva-aho, “Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms,” *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 2090–2095, 2012.

[19] P. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, “Weighted sum-rate maximization for a set of interfering links via branch and bound,” *IEEE Transactions on Signal Processing*, vol. 59, no. 8, pp. 3977–3996, 2011.

[20] P. Weeraddana, M. Codreanu, M. Latva-aho, A. Ephremides, and C. Fischione,”Weighted Sum-Rate Maximization in Wireless Networks: A Review,” *Foundations and Trends in Networking*, vol. 6, no. 1–2, pp. 1–163, 2012.

[21] B. Lee, L. Ngo, and B. Shim, “Antenna group selection-based user scheduling for massive MIMO systems,” in *IEEE Globecom’2014*, Dec. 2014, pp. 3302–3307.

[22] O. Mehanna, Nicholas D. Sidiropoulos, G. B. Giannakis, “Joint Multicast Beamforming and Antenna Selection”, IEEE Transactions on Signal Processing, vol, 61, No. 10, May 2013

[23] Makki, Behrooz; Ide, Anatole; Svensson, Tommy; Eriksson, Thomas; Alouini, Mohamed-Slim, “A Genetic Algorithm-based Antenna Selection Approach for Large-but-Finite MIMO Networks” IEEE Transactions on Vehicular Technology,2016

[24] A. F. Molisch, M. Z. Win, and J. H. Winters, “Capacity of MIMO systems with antenna selection,” in *Proc. IEEE Int. Conf. Commun.*, June 2001, pp. 570–574.

[25] T. Eng, N. Kong, and L. B. Milstein, “Comparison of diversity combining techniques for Rayleigh-fading channels,” *IEEE Trans. Commun.*, vol. 44, pp. 1117–1129, Sept. 1996.

[26] M. Z. Win and J. H. Winters, “Analysis of hybrid selection/maximalratio combining in Rayleigh fading,” *IEEE Trans. Commun.*, vol. 47, pp. 1773–1776, Dec. 1999.

[27] G.-T. Chyi, J. G. Proakis, and C. M. Keller, “On the symbol error probability of maximum-selection diversity reception schemes over a Rayleigh fading channel,” *IEEE Trans. Commun.*, vol. 37, pp. 79–83, Jan. 1989.

[28] E. A. Neasmith and N. C. Beaulieu, “New results on selection diversity,” *IEEE Trans. Commun.*, vol. 46, pp. 695–704, May 1998.

[29] Mohammad Gharavi-Alkhansari, and Alex B. Gershman, *Fast Antenna Subset Selection in MIMO Systems IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 52, NO. 2, FEBRUARY 2004*

[30] A. Gorokhov, “Antenna selection algorithms for MEA transmission systems,” *Proc. IEEE ICASSP*, pp. 2857–2860, May 2002.

[31] Yuanming Shi, Jun Zhang, and Khaled B. Letaief “Group Sparse Beamforming for Green Cloud-RAN” IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 13, NO. 5, MAY 2014

[32] A. Ebrahimi and A. Rahimian, “Estimation of channel parameters in a multipath environment via optimizing highly oscillatory error functions using a genetic algorithm,” in *Proc. 15th Int. Conf. SoftCOM*, Sep. 27–29, 2007, pp. 1–5.

[33] L. Hua, Z.Wei, Z. Qing-hua,W. Hua-kui, and Z. Zhao-xia, “Blind estimation of MIMO channels using genetic algorithm,” in *Proc. 5th Int. Conf. ICNC*, Aug. 14–16, 2009, vol. 4, pp. 163–167.

[34] S. Chen and Y. Wu, “Genetic algorithm optimisation for maximum likelihood joint channel and data estimation,” in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process.*, May 12–15, 1998, vol. 2, pp. 1157–1160.

[35] Z. Zhang, Y. Zhao, and J. Huang, “Array optimization for MIMO radar by genetic algorithms,” in *Proc. 2nd Int. Congr. CISP*, Oct. 17–19, 2009, pp. 1–4.

[36] A. Farkasvolgyi, R. Dady, and L. Nagy, “Channel capacity maximization in MIMO antenna system by genetic algorithm,” in *Proc. 3rd EuCAP*, Mar. 23–27, 2009, pp. 1119–1122.

[37] Y. Rahmat-Samii, “Modern antenna designs using nature inspired optimization techniques: Let Darwin and the bees help designing your multi band MIMO antennas,” in *Proc. IEEE Radio Wirel. Symp.*, Jan. 2007, pp. 463–466.

[38] Manuel O. Binelo, André L. F. de Almeida, *Member,* and F. Rodrigo P. Cavalcanti, MIMO Array Capacity Optimization Using a Genetic Algorithm, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, VOL. 60, NO. 6, JULY 2011

[39] Pramod Viswanath, *Member,*David N. C. Tse and Rajiv Laroia, “Opportunistic Beamforming Using Dumb Antennas” IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 48, NO. 6, JUNE 2002

[40] Cyril-Daniel Iskander and Hi-Tek Multisystems, “A MATLABR -based Object-Oriented Approach to Multipath Fading Channel Simulation” Formerly with The MathWorks, Natick, MA.

[41] Jeruchim, M. C., Balaban, P., and Shanmugan, K. S., Simulation of Communication Systems, Second Edition, New York, Kluwer Academic/Plenum, 2000.

[42] G. L. Turin *et al.*, “A statistical model of urban multipath propagation,” *IEEE Trans. Veh. Technol.*, vol. VT-21, no. 1, pp. 1–9, Feb. 1972.

[43] M. J. Gans, “A power-spectral theory of propagation in the mobile-radio environment,” *IEEE Trans. Veh. Technol.*, vol. VT-21, no. 1, pp. 27–38, Feb. 1972.

[44] http://cvxr.com/cvx/

[45] Z.-Q. Luo,W.Ma,A.So,Y. Ye, and S. Zhang, “Semidefinite relaxation of quadratic optimization problems,” *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

[46] M. Bengtsson and B. Ottersten, “Optimal and suboptimal transmit beamforming,” in Handbook of Antennas in Wireless Communications, L. C. Godara, Ed. CRC Press, 2001.

[47] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” IEEE Trans. Signal Process., vol. 55, no. 6, pp. 2646–2660, 2007.

[48] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, “Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms,” IEEE Trans. Signal Process., vol. 59, no. 3, pp. 1142–1157, 2011.

[49] Eric A. Jones and William T. Joines, Design of “Yagi-Uda Antennas Using Genetic Algorithms” IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 45, NO. 9, SEPTEMBER 1997

[50] S. Pillai, T. Suel, and S. Cha, “The Perron-Frobenius theorem: Some of its applications,” *IEEE Signal Processing Magazine*, vol. 22, no. 2, pp. 62-75, 2005.

[51] M. Schubert and H. Boche, “QoS-based resource allocation and transceiver optimization,” *Foundations and Trends in Communications and Information Theory*, vol. 2, no. 6, pp. 383-529, 2005.

APPENDIX A – SOURCE CODE FOR HEURISTIC BEAMFORMING

Heuristic Beamforming

```
function wMRT = functionMRT(H,D)
%Number of users
Kr = size(H,1);

%Total number of antennas
N = size(H,2);

%If D matrix is not provided, all antennas can transmit to everyone
if nargin<2
    D = repmat( eye(N), [1 1 Kr]);
end

%Pre-allocation of MRT beamforming
wMRT = zeros(size(H'));

%Computation of MRT, based on Definition 3.2
for k = 1:Kr
    channelvector = (H(k,:)*D(:,:,k))'; %Useful channel
    wMRT(:,k) = channelvector/norm(channelvector);%Normalization of useful
channel
end

function wZFBF = functionZFBF(H,D)
%Number of users
Kr = size(H,1);

%Total number of antennas
N = size(H,2);

%If D matrix is not provided, all antennas can transmit to everyone
if nargin<2
    D = repmat( eye(N), [1 1 Kr]);
end

%Pre-allocation of MRT beamforming
wZFBF = zeros(size(H'));

%Computation of ZFBF, based on Definition 3.4
for k = 1:Kr
    effectivechannel = (H*D(:,:,k))'; %Effective channels
    channelinversion = effectivechannel/(effectivechannel'*effectivechannel);
%Compute zero-forcing based on channel inversion
    wZFBF(:,k) = channelinversion(:,k)/norm(channelinversion(:,k));
%Normalization of zero-forcing direction
end



function wSLNRMAX = functionSLNRMAX(H,eta,D)
%Number of users
```

```
Kr = size(H,1);

%Total number of antennas
N = size(H,2);

%If eta vector is not provided, all values are set to unity
if nargin<2
    eta = ones(Kr,1)
end

%If D matrix is not provided, all antennas can transmit to everyone
if nargin<3
    D = repmat( eye(N), [1 1 Kr]);
end

%Pre-allocation of MRT beamforming
wSLNRMAX = zeros(size(H'));

%Computation of SLNR-MAX, based on Definition 3.5
for k = 1:Kr
    effectivechannel = (H*D(:,:,k))'; %Effective channels
    projectedchannel =
(eye(N)/eta(k)+effectivechannel*effectivechannel')\effectivechannel(:,k);
%Compute zero-forcing based on channel inversion
    wSLNRMAX(:,k) = projectedchannel/norm(projectedchannel);  %Normalization
of zero-forcing direction
end

function powerallocation = functionHeuristicPowerAllocation(rhos,q,weights)


Kt = size(rhos,1); %Number of base stations (BSs)
Kr = size(rhos,2); %Number of users (in total)


%Pre-allocation of matrix for power allocation coefficients
powerallocation=size(Kt,Kr);


%Iteration over base stations to perform power allocation
for j = 1:Kt
    indicesOfNonzero = find(rhos(j,:)>0); %Find which users that are served
by BS j
    nuAllActive =
(q(j)+sum(1./rhos(j,indicesOfNonzero)))/sum(weights(indicesOfNonzero));
    pk = sum(weights(indicesOfNonzero));
    pl = (1./rhos(j,indicesOfNonzero));
    nuRangeLower =
min(1./(rhos(j,indicesOfNonzero)'.*weights(indicesOfNonzero)));
    nuRangeUpper =
max(1./(rhos(j,indicesOfNonzero)'.*weights(indicesOfNonzero)));
    nu = fminbnd(@(x)
functionAllocDiff(x,q(j),rhos(j,indicesOfNonzero)',weights(indicesOfNonzero))
,nuRangeLower,nuRangeUpper);
```

```
    %Check if the difference between the allocated power and the available
    %power is minimized by allocating power to all users or only subset.
    if
functionAllocDiff(nu,q(j),rhos(j,indicesOfNonzero)',weights(indicesOfNonzero)
) <
functionAllocDiff(nuAllActive,q(j),rhos(j,indicesOfNonzero)',weights(indicesO
fNonzero))
        %Compute power allocation with optimal waterlevel (only a subset of
users are active)
        powerallocation(j,indicesOfNonzero) =
max([weights(indicesOfNonzero)*nu-1./rhos(j,indicesOfNonzero)'
zeros(length(indicesOfNonzero),1)],[],2);
    else
        %Compute power allocation with optimal waterlevel (all users are
active)
        powerallocation(j,indicesOfNonzero) =
max([weights(indicesOfNonzero)*nuAllActive-1./rhos(j,indicesOfNonzero)'
zeros(length(indicesOfNonzero),1)],[],2);
    end

    %Scale the power allocation to use full power (to improve numerical
accuracy)
    powerallocation(j,:) =
q(j)*powerallocation(j,:)/sum(powerallocation(j,:));
end
```

APPENDIX B – SOURCE CODE FOR OPTIMAL BEAMFORMING

Main Function

```
%%Simulation parameters
 rng('shuffle'); %Initiate the random number generators with a random seed


Nantennas = [4]; %Number of transmit antennas
K = 4; %Number of users
chrosomes = 500; % Number of intial chromosomes
numberofstrategies = 8; % Number of initial Population
J=5;
maximum_iteration = 200;
%Number of realizations in the Monte Carlo simulations
nbrOfMonteCarloRealizations = 2; %100;

%Combined channel matrix will be (K x K*N). This matrix gives the
%normalized variance of each channel element
channelVariances = [1 1 1 1];

%User weights for (unweighted) sum rate computation
weights = [1 1 1 1]'; ones(K,1);

%Range of SNR values
PdB = -5:5:30; %dB scale
P = 10.^(PdB/10); %Linear scale


  %Generate channel matrix for m:th realization
        H = repmat(sqrt(channelVariances)',[1 N]) .* Hall(:,:,m);

       %Compute normalized beamforming vectors for MRT
        wMRT = functionMRT(H);

        %Compute normalized beamforming vectors for ZFBF
        wZFBF = functionZFBF(H);

        %Go through all transmit powers
        for pind = 1:length(P)

            %Compute normalized beamforming vectors for transmit MMSE

            wSLNRMAX = functionSLNRMAX(H,P(pind)*ones(K,1));
            %Calculate power allocation with MRT
            rhos = diag(abs(H*wMRT).^2)';
            powerAllocationMRT =

            functionHeuristicPowerAllocation(rhos,P(pind),weights);

            %Calculate sum rate with MRT
            W = kron(sqrt(powerAllocationMRT),ones(N,1)).*wMRT;
```

```
    channelGains = abs(H*W).^2;
    signalGains = diag(channelGains);
    interferenceGains = sum(channelGains,2)-signalGains;
    rates = log2(1+signalGains./(interferenceGains+1));
    sumRateMRT(pind,m,n) = weights'*rates;


    %Calculate power allocation with ZFBF
    rhos = diag(abs(H*wZFBF).^2)';
    powerAllocationwZFBF =

    functionHeuristicPowerAllocation(rhos,P(pind),weights);

    %Calculate sum rate with ZFBF
    W = kron(sqrt(powerAllocationwZFBF),ones(N,1)).*wZFBF;
    channelGains = abs(H*W).^2;
    signalGains = diag(channelGains);
    interferenceGains = sum(channelGains,2)-signalGains;
    rates = log2(1+signalGains./(interferenceGains+1));
    sumRateZFBF(pind,m,n) = weights'*rates;


    %Calculate power allocation with transmit MMSE beamforming
    rhos = diag(abs(H*wSLNRMAX).^2)';
    powerAllocationwSLNRMAX_sumrate =
    functionHeuristicPowerAllocation(rhos,P(pind),weights);

    %Calculate sum rate with transmit MMSE beamforming
    W=
    kron(sqrt(powerAllocationwSLNRMAX_sumrate),ones(N,1)).*wSLNRMAX;
    channelGains = abs(H*W).^2;
    signalGains = diag(channelGains);
    interferenceGains = sum(channelGains,2)-signalGains;
    rates = log2(1+signalGains./(interferenceGains+1));
    sumRateMMSE(pind,m,n) = weights'*rates;



    %Save the rates of transmit MMSE beamforming to use as starting
    %point when calculating Optimal beamforming
    if computeOptimalBeamforming == true && P(pind)==P_BRB(pind_BRB)
        bestfeasibleRates(:,pind_BRB) = rates;
        pind_BRB = pind_BRB+1;
    end

 end

if computeOptimalBeamforming == true

        origin = zeros(K,1);

        %Computation of the utopia point using MRT, which is optimal
        %when there is only one active user
        utopia = zeros(K,1);
        for k = 1:K
```

```
                    utopia(k) = log2(1+abs(H(k,:)*wMRT(:,k))^2*P_BRB(pind));
                end

                bestfeasibleGenetic =
genetic_functionn(chrosomes,numberofstrategies,K,utopia',origin',
bestfeasibleRates(:,pind)',Qsqrt,weights,H,D,q,J,maximum_iteration);
                %Save the performance of the optimal beamforming
                sumrateOPTIMALgen(pind,m,n) = weights'*bestfeasibleGenetic';
            end

         end

     end
```

Genetic Function for Optimal Beamforming

```
function[bestfeasiblepointmutual]=genetic_function(numberofchromoses,posibles
trat,U,upperbound,lowerbound,bestfeasiblepoint,Qsqrt,weights,H,D,q,J,maximum_
iteration)

%%%chrosome creation%%%%%%
%N=15;
R = zeros(numberofchromoses,U);
L=1;

                %Normalized limit of the total transmit power
% q = ones(L,1);
 for k=1:U
     R(:,k) = lowerbound(k) + (upperbound(k)-
lowerbound(k)).*rand(numberofchromoses,1);
 end


%%%%%iterations start%%%%%%
%%%%% first step: initialization %%%%%
for ii = 1:maximum_iteration
%     for k=1:U
%     R(:,k) = lowerbound(k) + (upperbound(k)-
lowerbound(k)).*rand(numberofchromoses,1);
%     end
    S = posiblestrat;
    setsofrates=random_select(R,S,U);

    if(ii==1)
        for j=1:posiblestrat
            setsofrates_iteratives(j,:)=setsofrates(j,:);
        end
    else
        S = posiblestrat-J-1;
        for j=1:S
            setsofrates_iteratives(j,:)=setsofrates(j,:);
        end
        for l=1:posiblestrat-S
            setsofrates_iteratives(S+l,:)=bestfeasiblepoint_child(l,:);
        end
    end
```

```
%%%%second step: selection %%%%%

     setsofrates_iteratives;
    [value,pos] = queenset(setsofrates_iteratives,posiblestrat, weights);
    queenset_s = setsofrates_iteratives(pos,:);
    %%%check feasibility%%%%
    gammavar = 2.^(queenset_s(1,:))-1; %Transform lower corner into SINR
requirements
    [checkFeasibility,W] =
functionFeasibilityProblem_cvx(H,D,Qsqrt,q,gammavar); %Solve the feasibility
problem

       %Check if the point was feasible
       if checkFeasibility == false
             feasible(L) = 0 %Not feasible
             bestfeasiblepointmutual(1,:) = bestfeasiblepoint(1,:);
       elseif checkFeasibility == true
             feasible(L) = 1 %Feasible
             bestfeasiblepointmutual(1,:) = queenset_s(1,:)
       end

 %%%%third step: new generation child%%%%
    steps = 0;
    bestfeasiblepoint_child = zeros(J,U);
    for j=1:J
          steps = steps+0.01
          bestfeasiblepoint_child(j,:) = bestfeasiblepointmutual(1,:)+ steps
    end
    bestfeasiblepoint_child(J+1,:)= bestfeasiblepointmutual(1,:);
end



function[setsofrates] = random_select(R,K,U)
setsofrates = zeros(K,U);

for u=1:U
    % R([1:5 14:20 60:65 85:100 150:160],u) = 0;
     R([1:5 14:20 60:65 85:100],u) = 0;
end

 index_number = zeros(length(R),U);
 for u=1:U
    index_number(:,u) = randperm(length(R(:,u)));
 end
 %   setsofrats(:,:) = R(index_number(:,:));
% p=index_number(:,2);
 setsofallrates = zeros(length(R),U);
 for pp=1:U
     p=index_number(:,pp);
     setsofallrates(:,pp) = R(p(:),pp);
 end
 for k=1:K
     setsofrates(k,:)=setsofallrates(k,:);
 end
```

```
end
```

Feasibility Problem

```
function [feasible,Wsolution] =
functionFeasibilityProblem_cvx(H,D,Qsqrt,q,gammavar)

Kr = size(H,1); %Number of users
N = size(H,2); %Number of transmit antennas (in total)
L = size(Qsqrt,3); %Number of power constraints


%Solve the power minimization under QoS requirements problem using CVX
cvx_begin
cvx_quiet(true); % This suppresses screen output from the solver

variable W(N,Kr) complex;  %Variable for N x Kr beamforming matrix
variable betavar %Scaling parameter for power constraints

minimize betavar %Minimize the power indirectly by scaling power constraints

subject to

%SINR constraints (Kr constraints)
for k = 1:Kr

    %Channels of the signal intended for user i when it reaches user k
    hkD = zeros(Kr,N);
    for i = 1:Kr
        hkD(i,:) = H(k,:)*D(:,:,i);
    end

    imag(hkD(k,:)*W(:,k)) == 0; %Useful link is assumed to be real-valued

    %SOCP formulation for the SINR constraint of user k
    real(hkD(k,:)*W(:,k)) >= sqrt(gammavar(k))*norm([1 hkD(k,:)*W(:,[1:k-1
k+1:Kr])  ]);
end

%Power constraints (L constraints) scaled by the variable betavar
for l = 1:L
    norm(Qsqrt(:,:,l)*W,'fro') <= betavar*sqrt(q(l));
end

betavar >= 0; %Power constraints must be positive

cvx_end


%Analyze result and prepare the output variables.
if isempty(strfind(cvx_status,'Solved')) %Both power minimization problem and
feasibility problem are infeasible.
    feasible = false;
    Wsolution = [];
```

```
elseif betavar>1 %Only power minimization problem is feasible.
    feasible = false;
    Wsolution = W;
else %Both power minimization problem and feasibility problem are feasible.
    feasible = true;
    Wsolution = W;
end
```